Eidgenössische Finanzmarktaufsicht FINMA
Autorité fédérale de surveillance des marchés financiers FINMA
Autorità federale di vigilanza sui mercati finanziari FINMA
Swiss Financial Market Supervisory Authority FINMA

Technical description of the SST standard model reinsurance captive

Standard model insurance

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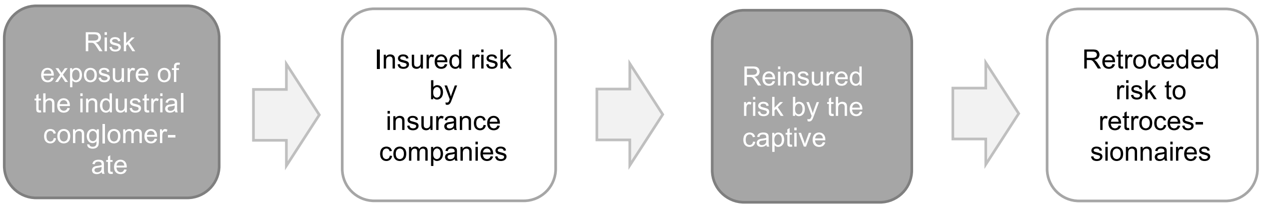
Contents

1 Aim

This technical description defines the standard model for reinsurance captives (in the sequel: *captive model*) according to art. 45 para. 1 ISO (*AVO/OS, SR/RS* 961.011) and is intended for reinsurance captives (in the sequel: *captives*) subject to the Swiss solvency test (SST). For the record of changes relative to the technical description of the SST standard model captive of previous year, see Section 13.

A captive reinsures normally only risks of its parent company, typically in an industrial conglomerate. There is a strong connection between them, e.g. detailed information about the ground losses are available to the captive and participation in a cash pool is likely. The following figure illustrates this, with dark grey coloured boxes being in the industrial conglomerate.



2 Scope

The captive model covers the non-life insurance risk of captives, which is illustrated by the dark gray box in a typical SST model, see figure below. For the modules for market risk, credit risk and aggregation please refer to the dedicated technical documentations on the SST page of the FINMA website[1].

Target capital (TC)						
Market risk	Credit risk		Insurance risk			Additional scenarios
	of other assets	of ceded retro	Life risk	Health risk	Non-life insurance risk	

[1] These can be downloaded at www.finma.ch > Supervision > Insurers > Cross-sectoral tools > Swiss Solvency Test (SST)

This technical description provides

- the description of the captive model;
- guidance on the positions of the SST balance sheet related to the assumed reinsurance and ceded retrocession business of the captive;
- formulas for calculating the market value margin;
- guidance on input coming from the captive model for the other modules, e.g. interest rate risk on the insurance cash flows and spread risks due to reinsurance and retrocession agreements in the market risk model.

Company-specific adjustments have to be adequately documented, especially adjustments that are not subject to prior approval in the sense of art. 9 para. 3 let. a ISO-FINMA (*AVO-FINMA/OS-FINMA; SR/RS* 961.011.1), see Section 11.1.4. The sections below provide more specific guidance where company-specific adjustments can be made.

3 Captives insurance business

Insurance business means here assumed (i.e. active) reinsurance and ceded (i.e. passive) retrocession of a captive. This section explains in which positions this is reflected in the SST balance sheet. A glossary for items related to captive business is provided. Moreover, a reporting requirement of the portfolio into a standardised segmentation is defined that is independent of the risk model.

3.1 SST balance sheet

The following indicates the positions in which the insurance business of a captive has to be reported in the SST balance sheet. See art. 3 ISO-FINMA for the scope of the SST balance sheet.

Name	Number in SST balance sheet	Position number (EHP-AVO-Kontenplan)	Description in the SST balance sheet of the corresponding position (in German)	Explanation
Assets				
Retro receivables	115)	110'200'100	Forderungen gegenüber Versicherungsgesellschaften: abgegebene	Receivables from retrocessionaires for ceded retrocession claims payments for already paid assumed reinsurance claims

Retro recoverables	101) 102) 103)	106'203'000	Aktive Rückversicherung: Schadenversicherungsgeschäft *Aktive Rückversicherung (Schaden) - verdientes Geschäft* *Aktive Rückversicherung (Schaden) - unverdientes Geschäft*	Receivables from retrocessionaires for ceded retrocession claims payments for not yet paid assumes reinsurance claims: • *earned* • *unearned*
Reinsurance premium receivables	116)	110'200'200	Forderungen gegenüber Versicherungsgesellschaften: übernommene	Receivables from cedants (or intermediaries) for assumed reinsurance premium payments
Reinsurance deposits	86)	104'000'000	1.4 Depotforderungen aus übernommener Rückversicherung	Receivables from cedants (or intermediaries) for assumed reinsurance premium deposits

Liabilities				
Reinsurance provisions (reserves)	151) 152) 153) 154)	201'203'000	Aktive Rückversicherung: Schadenversicherungsgeschäft Aktive Rückversicherung: Best Estimate der Versicherungsverpflichtungen (Schaden) - verdientes Geschäft Aktive Rückversicherung: Best Estimate der Versicherungsverpflichtungen (Schaden) - unverdientes Geschäft Aktive Rückversicherung: Best Estimate der sonstigen Versicherungsverpflichtungen (Schaden)	Obligations towards cedants for assumed reinsurance claims payments (gross): • *earned* • *unearned* • *other*
Retro premium liabilities	189)	207'000'000 excl. 207'300'200	Sonstige Verbindlichkeiten aus dem Versicherungsgeschäft	Obligations towards retrocessionaires for ceded retrocession premium payments
Retro deposit liabilities	186)	206'000'000	2.6 Depotverbindlichkeiten aus abgegebener Rückversicherung	Obligations towards retrocessionaires for ceded retrocession premium deposits

3.2 Glossary

We define the following terms for the insurance business of a captive in alphabetical order:

- **Accident year (occurrence year)**: losses occurring (and premiums earned) in the corresponding calendar year. This is (only) from assumed reinsurance contracts that are in force for some period in the calendar year and may include contracts on risk attaching basis and on loss-occurring basis.
  - Current accident year (CY) refers to the period from t_0 (excluded) to t_1 (included)
  - Prior accident years (PY) refers to the period until t_0
- **Earned business**: existing business with losses having occurred in the prior accident years
- **Existing business**: business incepted until t_0
- **Gross loss**: losses ceded to captive gross of ceded retrocession
- **Inception/incepting:** refers to the coverage period in the sense of art. 3 para. 5 ISO-FINMA
- **Assumed reinsurance (or inward or active reinsurance)**: refers to all assumed business of a captive
- **LOB**: line of business
- **Material/materiality**: as defined in art. 42 al. 2 ISO
- **Net loss**: losses ceded to captive net of ceded retrocession
- **New business**: business incepting between t_0 (excluded) and t_1 (included)
- **Outstanding losses**: includes all outstanding loss payments, regardless of whether they are reported or not ("ultimate view"). Includes in particular case reserves, ACR (additional case reserves), IBNyR (incurred but not yet reported), IBNER (incurred but not enough reported) and the corresponding claims adjustment expenses (ULAE and ALAE).
- **Ceded retrocession (or outward or passive retrocession)**: this term is used to refer to all passive reinsurance protection of a captive (i.e. retrocession)
- **SST currency**: as defined in art.4 ISO-FINMA. (In order to use a currency other than the provided default SST currencies, the captive must apply for an adjustment subject to approval according to in the sense of art. 9 para. 3 let. a ISO-FINMA.)
- t_0 (or $t = 0$): **reference date** of the SST calculation
- t_1 (or $t = 1$): **end of the one-year period** starting from the reference date
- **Underwriting year**: losses arising (and premiums) from any assumed reinsurance contract written (i.e. incepting) in the corresponding one-year period. This may include contracts on risks attaching basis and on losses occurring basis. (Note that the SST balance sheet and the modelling are supposed to be in line with art. 3 para. 5 OS-FINMA.)
- **Unearned business**: existing business with losses occurring in the current or future accident years
- **Ultimate losses (at time t)**: sum of cumulated paid at time t and outstanding losses
  - **Best estimate of ultimate losses (at time t)**: sum of cumulated paid at time t and best estimate of outstanding losses conditional the available information at time t
  - **Final ultimate losses** (at time t): cumulated paid losses at time t when the best estimate of outstanding losses conditional the available information at time t equals 0, i.e. settled losses

3.3 Reserves and premiums reporting requirement

Net reserves (reinsurance reserves minus retro recoverables) at t_0 and net expected premiums have to be reported in the prescribed reporting segmentation, possibly through appropriate proxies, as published in the SST-Captive-Template[2]. Note that here the reporting of undiscounted values is requested.

4 The one-year change in risk-bearing capital

4.1 Introduction to cash flows and best estimates

4.1.1 Discounting

The SST reference time for discounting is t_0, the start of the current year. The risk-free yield curve specified by FINMA for SST currency (see art. 31 para. 1 ISO) is denoted by $\{r_k\}_{k\geq 1}$. For instance, r_2 denotes the annual risk free interest rate for maturity at time t_2 starting from time t_0 and indicates a two-year term. Accordingly, the corresponding discount factors from t_k to t_0 are written

$$v_0 := 1,\ v_1 = \frac{1}{1+r_1}, v_2 = \frac{1}{(1+r_2)^2}, \ldots, v_k = \frac{1}{(1+r_k)^k}, \ldots$$

Further, we extend the notation of interest rates by writing $R_{j,s}$ for $0 \leq j < s$: it means maturity at time t_s starting from time t_j, a term of $k = s - j$ years. Capital letters emphasise the stochasticity viewed from t_0 for $j > 0$ and for instance $R_{1,3}$ denotes the two-year term risk free interest rate starting from t_1 until maturity t_3. For $j = 0,\ k = s$ and $R_{0,k} = r_k$ is deterministic.

We assume (as simplifying independence assumption on interest rates) the following relations:

$$E\big((1+R_{1,k+1})^k\big) = \frac{(1+r_{k+1})^{k+1}}{1+r_1}$$

Writing and assuming $v_{1,k+1} = 1/E\big((1+R_{1,k+1})^k\big) \approx E\big(1/(1+R_{1,k+1})^k\big)$ for the expected discount factors from time t_{k+1} to time t_1, it follows that:

$$v_{1,2} = \frac{v_2}{v_1}, v_{1,3} = \frac{v_3}{v_1}, \ldots v_{1,k+1} = \frac{v_{k+1}}{v_1}, \ldots$$

The above can for example be used for discounting of a cash flow $CF^{(t_1)} = \{CF_2, CF_3, \ldots\}$ outstanding at time t_1 (we will show two cases: discounting to t_1 and discounting to t_0). In this notation the superscript (t_1) indicates that future cash flows after t_1 are considered. The sequence begins with CF_2 which represents the sum of all cash-inflows and cash-outflows between t_1 and

[2] This corresponds exacly to the "RE_reserves_and_premiums" sheet in the SST-StandRe-Template, see the technical description for the SST standard model reinsurance (StandRe), Section 3. The prescribed reporting segmentation defines LoBs, geographical regions and types of contracts.

t_2. It is assumed as a simplification that cash-inflows and cash-outflows occur at the end of the respective time interval.

Assuming deterministic discounting, the value of this total cash flow discounted to time t_1 is given by:

$$v_{1,2} \cdot CF_2 + v_{1,3} \cdot CF_3 + \cdots = \sum\nolimits_{k \geq 1} \frac{v_{k+1}}{v_1} \cdot CF_{k+1}$$

and the same total cash flow discounted to time t_0 is given by

$$v_2 \cdot CF_2 + v_3 \cdot CF_3 + \cdots = \sum\nolimits_{k \geq 1} v_{k+1} \cdot CF_{k+1}$$

4.1.2 Best estimate of a cash flow

We denote with $CF^{(u)} = \{CF_{u+1}, CF_{u+2}, \dots\}$ an undiscounted cash flow outstanding at time $u \geq 0$ (by convention $u = 0$ meaning t_0, $u = 1$ meaning t_1, etc.). For a given time $t \geq 0$, we define $v_{t,s} = \frac{v_s}{v_t}$ for $s \geq 0$ as the discount factor from time s to time t and write $\mathcal{F}_t$ for the information known until time t.

We denote by

$$BE_t^{(u)} = BE_t\left(CF^{(u)}\right)$$

the best estimate at time t of an underlying cash flow CF outstanding at time u, discounted to time t_0, defined by:

$$BE_t^{(u)} = v_t \cdot E\left(\sum_{k \geq 1+u-t} v_{t,t+k} \cdot CF_{t+k} \middle| \mathcal{F}_t \right) = E\left(\sum_{k \geq 1+u-t} v_{t+k} \cdot CF_{t+k} \middle| \mathcal{F}_t \right) = E\left(\sum_{j \geq 1+u} v_j \cdot CF_j \middle| \mathcal{F}_t \right)$$

Note that we use discounting to time t_0 in our definition, see factor v_t at the first equality above. If $u = t$ and the underlying cash flow is for business in scope of the balance sheet at time t, this provides as a special case the classical best estimate reserves in the SST balance sheet at time t:

$$BE_t^{(t)} = E\left(\sum_{k \geq 1} v_{t+k} \cdot CF_{t+k} \middle| \mathcal{F}_t \right) = E\left(\sum_{j \geq t+1} v_j \cdot CF_j \middle| \mathcal{F}_t \right)$$

and if $u < t$, one has a decomposition into cash flows between u and t and the corresponding reserves in the SST balance sheet at time t:

$$BE_t^{(u)} = \sum_{j=u+1}^{t} v_j \cdot CF_j + BE_t^{(t)}$$

If the undiscounted cash flow $CF^{(u)} = \{CF_{u+1}, CF_{u+2}, \ldots\}$ is further decomposed into N_{seg} segments (e.g. lines of business) by using the superscript m for a segment, i.e. $CF^{m,(u)} = \{CF^m_{u+1}, CF^m_{u+2}, \ldots\}$ for $m = 1, \ldots, N_{seg}$ and $CF_{u+k} = \sum_{m=1}^{N_{seg}} CF^m_{u+k}$ for $k \geq 1$, then the corresponding discounted best estimates add up:

$$BE_t^{(u)} = \sum_{m=1}^{N_{seg}} BE_t^{m,(u)}$$

and for each m it would be preferable to use discount factors of the currency underlying the segment m, which can possibly differ from the SST currency.

Where an undiscounted best estimate is needed, we use the notation (N) for "nominal" in superscript $BE_t^{(N),(u)} = BE_t^{(N)}\left(CF^{(u)}\right)$ and this is defined by setting all v_js to one in the above formulas.

The risk resulting from the stochasticity of the yield curves is modelled in the market risk module of the SST, see also section 10 in this document. Within the insurance risk we assume deterministic discounting, i.e. we consider expected cash flow patterns and expected yield curves as basis for the discount factors. The interest rate risk is solely considered in the market risk module.

Unless otherwise stated, in the following sections amounts are deterministically discounted to time t_0 (the reference date of the SST calculation) and expressed in SST currency.

4.2 One-year change in scope

The captive model quantifies the one-year change (from time t_0 to time t_1) in the risk-bearing capital related to non-life insurance risk, assuming the *simplifications for the one-year change and the formula for the target capital* in Section 3.2 of the technical description for the SST standard model aggregation and market value margin.

Remark: The target capital is given by the **negative** of the Expected Shortfall ES_α, where the Expected Shortfall corresponds in the continuous case to the mean of the α lowest outcomes, with $\alpha \ll 1$. In this representation, losses are negative numbers. For the parts of the captive model where losses S are represented as positive numbers, we define the right-hand expected shortfall as $ES^{1-\alpha}(S) \coloneqq -ES_\alpha(-S)$. The following properties hold:

i. If $S \leq T$, $ES^{1-\alpha}(T) \geq ES^{1-\alpha}(S)$;

ii. $ES^{1-\alpha}(S + T) \leq ES^{1-\alpha}(S) + ES^{1-\alpha}(T)$;

iii. $ES^{1-\alpha}(a \cdot S) = a \cdot ES^{1-\alpha}(S)$ if $a > 0$; and

iv. $ES^{1-\alpha}(S + a) = ES^{1-\alpha}(S) + a$ for $a \in \mathbb{R}$.

The scope of the target capital for insurance risk consists of all insurance-related cash flows outstanding at time t_0 for the business assumed by the captive which incept until time t_1. The cash flow consists of all relevant premiums, losses and expenses net of ceded retrocession and is in line art. 3 para. 5 ISO-FINMA.

We use the notation " $\rightarrow t_0$ " for the business incepted until time t_0 and " $\rightarrow t_1$ " for the business incepted until time t_1 .

Writing $BE_t^{\rightarrow s,(u)} = BE_t\left(CF^{\rightarrow s,(u)}\right)$ for the discounted best estimate at time t of an underlying cash flow CF outstanding at time u for the business incepted until time s, the one-year change in the risk-bearing capital related to non-life insurance risk is expressed as:

$$BE_{t_1}^{\rightarrow t_1,(t_0)} - BE_{t_0}^{\rightarrow t_0,(t_0)} = BE_{t_1}^{\rightarrow t_1} - BE_{t_0}^{\rightarrow t_0}$$

omitting $(u) = (t_0)$ in the notation on the right-hand side of the equation.

Remark: Using $BE_{t_1}^{\rightarrow t_1,(t_0)} = v_1 \cdot CF_{t_1}^{\rightarrow t_1} + BE_{t_1}^{\rightarrow t_1,(t_1)}$ it can be shown that this change corresponds to the one year change in risk bearing capital due to insurance risk.

Writing further " $t_0 \rightarrow t_1$ " for the business incepted between t_0 and t_1 (referred to also as new business), the above one-year change can be decomposed into a term that represents the one-year change in best estimate for the same cash flows and the same business incepted until t_1 (whence, by definition of best estimate, with mean zero i.e. centered), and another term that represents the expected non-life insurance result of the new business, as follows:

$$BE_{t_1}^{\rightarrow t_1} - BE_{t_0}^{\rightarrow t_0} = BE_{t_1}^{\rightarrow t_1} + BE_{t_0}^{t_0 \rightarrow t_1} - BE_{t_0}^{t_0 \rightarrow t_1} - BE_{t_0}^{\rightarrow t_0} = \left(BE_{t_1}^{\rightarrow t_1} - BE_{t_0}^{\rightarrow t_1}\right) + BE_{t_0}^{t_0 \rightarrow t_1}$$

where we define the term $BE_{t_0}^{t_0 \rightarrow t_1}$ as

- *expected non-life insurance result* = net expected premiums minus net expected discounted losses minus expected expenses of the new business.

The captive model assumes that premiums and operating expenses are deterministic or can be modelled deterministically. Loss dependent expenses (e.g. ALAE, ULAE, sliding scale commissions, etc.) should be modelled within the losses. Thus, the remaining of the captive model provides a distribution for $Z^{NL\text{-}insurance\text{-}risk}$ defined as

- *centered non-life insurance risk* = one-year change in the best estimate of the cash flow arising from losses only, equals $BE_{t_1}^{\rightarrow t_1} - BE_{t_0}^{\rightarrow t_1}$.

The distribution reflects events that occur during the current one-year period, i.e. between t_0 and t_1. The distribution models new claims, but also takes into account any new information like court decisions that may have an impact on the business in scope.

Unless otherwise stated, distributions and expectations are meant to be related to the information available at time t_0.

Note that the word "risk" refers to different objects according to the context: a random variable; the corresponding distribution; the corresponding stand-alone capital requirement from the expected shortfall at the prescribed confidence level $1 - \alpha$.

The captive model provides separate distributions for reserve risk and premium risk and gives assumptions for their aggregation.

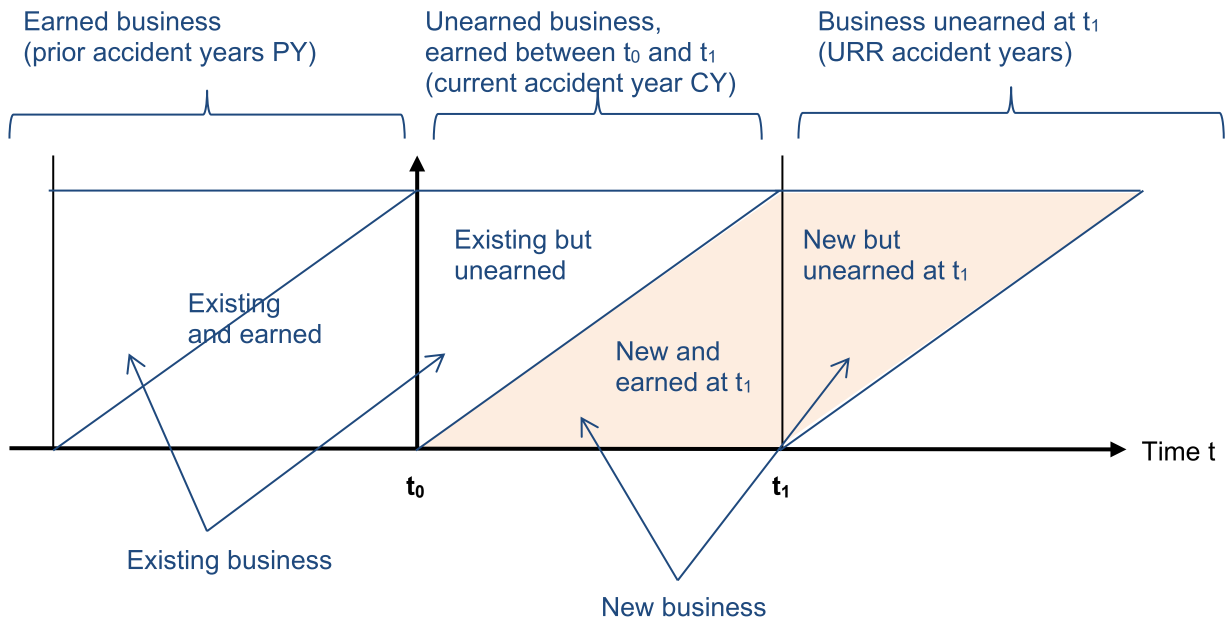
More precisely, we first introduce a partition of the business in scope, i.e. incepted until t_1. Then we explain how this is related to the one year change $Z^{NL\text{-}insurance\text{-}risk}$. In this way we introduce proper definitions of reserve risk and of premium risk, respectively. The latter is again split into CY risk and URR risk.

4.3 Partition of the business

The partition of the *business incepted until* t_1 in five sets denoted $\{"ex\text{-}e"; "ex\text{-}u\text{-}e_1"; "ex\text{-}u_1"; "new\text{-}e_1"; "new\text{-}u_1"\}$ is as follows:

- *existing, earned* ***(ex-e)***: business incepted until t_0 and earned at time t_0;
- *existing, unearned* ***(ex-u)***: business incepted until t_0 and unearned at time t_0. This will be earned at a latter time and thus can be further split into an earned at t_1 and unearned at t_1 parts:
  - existing, unearned, earned(t_1) **(ex-u-e$_1$)**; and
  - existing, unearned, unearned(t_1) = existing, unearned(t_1) **(ex-u$_1$)**
- *new:* business incepted between t_0 and t_1. Can be further split into:
  - new, earned(t_1) **(new-e$_1$)**; and
  - new, unearned(t_1) **(new-u$_1$)**.

This is illustrated for typical one-year contracts in the following representation:



4.4 Definition of reserve risk and premium risk

From the partition above, by linearity of expectations and assuming implicitly linear and unbiased related estimators, the following decomposition of the one-year change holds:

$$
\begin{aligned}
Z^{NL\text{-}insurance\text{-}risk} &= BE_{t_1}^{\to t_1} - BE_{t_0}^{\to t_1} = \left(BE_{t_1}^{ex-e} - BE_{t_0}^{ex-e}\right) \\
&+ \left(BE_{t_1}^{ex-u-e_1} - BE_{t_0}^{ex-u-e_1}\right) + \left(BE_{t_1}^{new-e_1} - BE_{t_0}^{new-e_1}\right) \\
&+ \left(BE_{t_1}^{ex-u_1} - BE_{t_0}^{ex-u_1}\right) + \left(BE_{t_1}^{new-u_1} - BE_{t_0}^{new-u_1}\right)
\end{aligned}
$$

From this, we define

- $Z^{reserve\text{-}risk} = \left(BE_{t_1}^{ex-e} - BE_{t_0}^{ex-e}\right)$, the reserve risk (also named PY risk); and
- $Z^{premium\text{-}risk} = Z^{CY\text{-}risk} + Z^{URR\text{-}risk}$, the premium risk, where
  - $Z^{CY\text{-}risk} = \left(BE_{t_1}^{ex-u-e_1} - BE_{t_0}^{ex-u-e_1}\right) + \left(BE_{t_1}^{new-e_1} - BE_{t_0}^{new-e_1}\right)$ defines the CY risk, and
  - $Z^{URR\text{-}risk} = \left(BE_{t_1}^{ex-u_1} - BE_{t_0}^{ex-u_1}\right) + \left(BE_{t_1}^{new-u_1} - BE_{t_0}^{new-u_1}\right)$ defines the URR risk

PY stands for *previous year*, CY for *current year* and URR for *unexpired risk reserve*.

Remark 1: $E\left(BE_{t_1}^{\to t_1}\right) = BE_{t_0}^{\to t_1}$ and the same holds for e.g. $E\left(BE_{t_1}^{ex-e}\right) = BE_{t_0}^{ex-e}$.

Remark 2: The decomposition above plus the expected non-life insurance result is the formula (21) of the technical description for the SST standard model non-life insurance. The following table provides the correspondence:

Above formula of technical description for the SST standard model captives (this document)	Formula (21) of technical description for the SST standard model non-life insurance
expected non-life insurance result term excluded	expected non-life insurance result term included
deterministic discounting	stochastic discounting
implicit discount factors (representation of discounted amounts)	discount factors shown explicitly and representation of undiscounted amounts
symbols for cash flows outstanding at t_0	where applicable, decomposition into cash-in and cash-out between t_0 and t_1 and cash flows outstandings at t_1
$ex\text{-}e$	PY, t_0
$ex\text{-}u\text{-}e_1$	$CY, Bestand$
$new\text{-}e_1$	CY, Neu
$ex\text{-}u_1$	URR, t_0
$new\text{-}u_1$	URR, Neu

4.5 Currencies

The standard currencies of the captive model are CHF, EUR, USD, GBP and JPY.

For other currencies, the mapping to the standard currencies has to be consistent with the mapping used in the SST standard model market risk.

If the captive needs to use its own currency, it has to apply for an adjustment subject to approval in the sense of art. 9 para. 3 let. a ISO-FINMA.

5 Reserve risk model

The captive model assumes that the reserve risk component of the decomposition given at Section 4.4 can be further decomposed as a sum of random variables, i.e. $Z^{reserve\text{-}risk} = -X^{PY} - \epsilon^{PY}$. The minus sign is on purpose, because we work in this section with loss variables (a loss is represented by a positive value). X^{PY} represents a loss resulting from the lognormal distribution of the reserve risk model below, whereas ϵ^{PY} represents the part of the reserve risk that is not or not sufficiently modelled by the reserve risk model. ϵ^{PY} can be partially modelled by individual events, see Section 7. The aggregation to non-life insurance risk, including the dependency assumptions and the non-modelled remaining part, is explained in Section 8. X^{PY}results from net cash flows after retrocession recoverables if applicable.

Writing $X^{PY} = BE_{t_1}(Y^{PY,t_0}) - BE_{t_0}(Y^{PY,t_0})$ with Y^{PY,t_0} denoting outstanding losses at time t_0 for the business incepted in previous years and earned until t_0, the captive model assumes that $BE_{t_1}(Y^{PY,t_0})$ is modelled by a log-normal distribution. See Section 12.2 in Appendix for properties of the log-normal distribution.

With $BE_{t_1}(Y^{PY,t_0}) = d^{PY} \cdot BE_{t_1}^{(N)}(Y^{PY,t_0})$, it follows that the distribution of $BE_{t_1}(Y^{PY,t_0})$ is specified by the discount factor d^{PY}, and by the mean μ and the coefficient of variation CV of the undiscounted $BE_{t_1}^{(N)}(Y^{PY,t_0})$.

The captive defines an appropriate set of segments regarding its earned business, the previous years' parameter segments, providing the decomposition $Y^{PY,t_0} = \sum_{m \in PY\text{-}seg} Y_m^{PY,t_0}$. We denote the mean and the coefficient of variation of $BE_{t_1}^{(N)}\left(Y_m^{PY,t_0}\right)$ by μ_m and CV_m, resp., and the components of the related correlation matrix by Γ_{ij}. It suffices to provide estimates to μ_m, CV_m and Γ_{ij} to obtain μ and CV by moment aggregation, see Section 12.1 in Appendix.

μ_m is equal to the $BE_{t_0}^{(N)}\left(Y_m^{PY,t_0}\right)$, i.e. the net reserves for earned business at t_0 related to parameter segment m. However, no reserve risk needs to be allocated for claims that are reserved in the SST balance sheet up to the full contract limit without the possibility of further deterioration . In other words, the input value $BE_{t_0}^{(N)}\left(Y_m^{PY,t_0}\right)$ of the reserve risk component can be reduced by the share of the reserves meeting the conditions given above.

The correlation between parameter segments is set to 0.5, i.e. $\Gamma_{ij} = 0.5$ for $i \neq j$ and $\Gamma_{ij} = 1$ else.

The coefficient of variation CV_m is set to 15% for each parameter segment. Alternatively, a company can or might even have to use its own CVs if, when using the standard CV, the capital requirement is not sufficient in relation to the risk situation. In this case, own CVs have to be derived and applied to each and every parameter segment. The choice of the individual parameters shall be justified. Own CVs may be based on gross rather than net data for simplification.

The discounting in the parameter segment m, typically to compute the discounted value $BE_{t_0}\left(Y_m^{PY,t_0}\right)$ from its undiscounted value μ_m , is done by the deterministic discount factor

$$d_m^{PY} = \sum_{k\geq 1} \nu_k \cdot \pi_k^{PY,m}$$

where $\pi_1^{PY,m}, \pi_2^{PY,m}, \ldots$ is an expected incremental payment pattern ($\sum_{k\geq 1} \pi_k^{PY,m} = 1$) provided by the captive. Payments are expected to be in the underlying currency of the parameter segment, which may differ from the SST currency. Accordingly, discount factors ν_k should be those of the currency of the parameter segment. However, if this simplification is not material, discount factors corresponding to the SST currency can also be used. The discount factor for all PY parameter segments can be obtained by

$$d^{PY} = \frac{\sum_{m\in PY-seg} d_m^{PY} \cdot \mu_m}{\sum_{m\in PY-seg} \mu_m}$$

The reserve risk module in the captive model uses the value of the net reserves. It is not based on explicit ground-up simulations and/or the explicit modelling of reinsurance conditions. If the risk situation requires to model the reserve risk in an alternative way, the captive has to apply for a company-specific adjustment in the sense of mn 107-109 FINMA Circ. 17/3 "SST".

6 Premium risk model

6.1 Introduction

The premium risk model is a model for the current year risk component of the decomposition given at Section 4.4. Thus, similar to reserve risk, the captive model assumes a decomposition $Z^{CY\text{-}risk} = -X^{CY} - \epsilon^{CY}$ and $Z^{URR\text{-}risk} = -\epsilon^{URR}$, where X^{CY} represents a loss coming from the distribution of the CY risk model below, ϵ^{CY} represents the part of the current year risk that is not or not sufficiently modelled with X^{CY}, and ϵ^{URR} emphasises that no specific model is defined for the URR risk. X^{CY} results from net cash flows after retrocession recoverables, if applicable. Nonetheless,

- If URR risk is material, the following rules can be applied within the CY risk model: artificially expand the frequency of a parameter segment or duplicate the parameter segment into a CY and a URR segment or alternatively, if applicable, into two segments based on underwriting year (e.g. 1 January CY to 30 June CY, and 1 July CY to 30 June CY+1). If the URR risk cannot be modelled sufficiently by applying these rules the captive needsto apply for an adjustment subject to approval in the sense of art. 9 para. 3 let. a ISO-FINMA;

- Gross instead of net modelling (of ceded retrocession) is possible if it is conservative. Gross modelling means that reinsurance conditions are applied only to assumed reinsurance contracts.

The captive defines an appropriate set of segments regarding its unearned business, the current year parameter segments, providing the decomposition $X^{CY} = \sum_{m \in CY\text{-}seg} X_m^{CY}$. The CY parameter segments may differ from the PY parameter segments if necessary. The captive model assumes independence between CY parameter segments.

We write $X_m^{CY} = BE_{t_1}\left(\tilde{Y}_m^{CY,t_0}\right) - BE_{t_0}\left(\tilde{Y}_m^{CY,t_0}\right)$ with $\tilde{Y}_m^{CY,t_0}$ denoting outgoing cash flows (as positive values) outstanding at time t_0, for business earned between t_0 and t_1 (current accident year CY made of unearned existing business and new business), in the parameter segment m. For CY, the captive model assumes modelling of ultimate instead of best estimate at time t_1 , that is $X_m^{CY} \approx d_m^{CY} \cdot \left(\tilde{Y}_m^{CY,t_0} - BE_{t_0}^{(N)}\left(\tilde{Y}_m^{CY,t_0}\right)\right)$ where the discount factor d_m^{CY} is defined similar to d_m^{PY} but with the corresponding payment pattern for new claims. For CY risk two approaches are available in the captive model (the second being a simplified conservative approach):

- ground-up modelling based on ground-up claims of the parent company, for deriving the distribution of $\tilde{Y}_m^{CY,t_0}$ (see section 6.2); or
- maximal possible loss (MPL) modelling (see section 6.3).

6.2 Ground-up modelling

By ground-up modelling CY claims, the company uses a transformation function from ground-up claims to captive claims, including conditions across several parameter segments if necessary. Frequency-severity models by CY parameter segment m are assumed for ground up claims, and then the transformation modelling the assumed reinsurance and ceded retrocession is applied to obtain the desired distribution of captive claims. The following applies by parameter segment; we omit the index m of the parameter segment.

6.2.1 Ground-up claims model

The ground-up claims consist of attritional claims or large claims, both beeing modelled by a frequency-severity approach:

- attritional claims are assumed to have a Poisson-distributed frequency N^a and a Gamma-distributed undiscounted severity Y^a;
- large claims are assumed to have a Poisson-distributed frequency N^l and a Pareto-distributed undiscounted severity Y^l;
- all random variables N^a, Y_1^a, Y_2^a, ..., N^l, Y_1^l, Y_2^l, ... are independent.

Thus the corresponding ground-up parameters have to be estimated by the captive:

- the frequency $E(N^a)$ of attritional claims which can be estimated by the average number of historical attritional losses. The same applies mutatis mutandis to the frequency of large claims $E(N^l)$. Expected changes in the frequency due to risk of change (e.g. exposure changes, legislative changes, behavioural changes, etc.) need to be taken into account;
- the expected severity of attritional claims $E(Y_i^a)$ which can be estimated from the average amount of historical attritional claims. The corresponding standard deviation $sd(Y_i^a)$ is obtained in a similar way;
- the large loss threshold x_0 and Pareto shape α which can be obtained based on data and using actuarial expert judgement.

It might be the case that historical attritional claims (paid and incurred) and the number of attritional claims are only available on an annual aggregate basis. This means that only the historical realisations of the random variables $Y^a = \sum_{i=1}^{N^a} Y_i^a$ and N^a are available. In that case the following procedure can be applied:

- The expected number of claims $E(N^a)$ together with the mean $E(Y^a)$ and standard deviation $sd(Y^a)$ of the aggregate claim amount can be estimated.
- The mean $E(Y_i^a)$ and the standard deviation $sd(Y_i^a)$ of a single loss are derived by using the following formulas: $E(Y^a) = E(N^a) \cdot E(Y_i^a)$ and $Var(Y^a) = E(N^a) \cdot Var(Y_i^a) + E(Y_i^a)^2 \cdot Var(N^a)$.

Historical data used for the estimation of parameters have to be adjusted such that they are comparable over time. For instance, incurred losses should be adjusted according to the corresponding inflation. When historical data are insufficient to obtain reliable estimates, the captive explains how the estimate was derived and justifies it.

The frequency distribution is assumed to be Poisson for $N = N^a$ or $N = N^l$; in particular $Var(N) = E(N)$. In case of over-dispersion (i.e. $Var(N) > E(N)$, which is possibly the case if the number of losses is large), the frequency can be modelled by a negative binomial distribution instead of a Poisson distribution, or must be if the impact on solvency is material. By using a negative binomial distribution, the variance $Var(N)$ needs to be estimated, too.

If the expected frequency of attritional claims is high and might cause a long run-time for the simulation, the frequency-severity model can be replaced by an aggregated claims distribution using the following property:

- The sum of N independent identically distributed Gamma distributed random variables is Gamma distributed. The mean and the standard deviation of the distribution of the aggregate claim can be deducted from the expected frequency f, the mean m and the standard deviation s of the single claims. They are given, respectively, by $f \cdot m$ and $\sqrt{f} \cdot s$.

In the captive model, it is possible to replace the frequency-severity model by an expected frequency of 1 and a severity using the expectation and standard deviation of the aggregate above. Note, however, that applying on the aggregate ground-up claim transformations to the captive defined on single claims (see below) yields incorrect results. This simplification can be accepted if the impact on solvency is low.

The replacement of values is used by default when the estimated expected frequency in the input is greater than 10, see Section 11.1.7. Not replacing values in the frequency-severity model remains allowed as alternative.

The model above is used with simulations. One simulation year for the underlying parameter segment m and regarding ground-up claims is depicted as follows, with $r.Distribution$ denoting a random draw from the underlying $Distribution$; for the parametrisation of the Gamma distribution, see Section 12.3 in Appendix.

Attritional ground-up claims
$E(N^a) \;\rightarrow\; \lambda = E(N^a) \;\rightarrow\; r.Poisson(\lambda) \;\rightarrow\; n_a$
$E(Y^a),\ \sigma(Y^a) \rightarrow k = \frac{E(Y^a)}{sd(Y^a)},\ \theta = \frac{[sd(Y^a)]^2}{E(Y^a)} \rightarrow r.Gamma(k,\theta)\ \ [n^a \text{ independent draws}] \;\rightarrow\; y_1^a, \dots, y_{n_a}^a$
or if $E(N^a) > 10$:
$E(Y^a),\ \sigma(Y^a) \;\rightarrow\; k = E(N^a) \cdot E(Y^a),\ \theta = \sqrt{E(N^a)} \cdot sd(Y^a) \;\rightarrow\; r.Gamma(k,\theta) \;\rightarrow\; y_1^a = y_{agg}^a$

Large ground-up claims
$E(N^l) \;\rightarrow\; \lambda = E(N^l) \;\rightarrow\; r.Poisson(\lambda) \;\rightarrow\; n_l$
$x_0\,,\ \alpha \;\rightarrow\; r.Pareto_{x_0}(\alpha)\ \ [n^l \text{ independent draws}] \;\rightarrow\; y_1^l, \dots, y_{n_l}^l$

6.2.2 Transformation into captive claims

The undiscounted net claims $\tilde{Y}^{CY}$ of the reinsurance captive are obtained by applying the assumed reinsurance and ceded retrocession structures to the ground-up claims. These structures can be modelled by a transformation $f\colon \mathbb{R}^{\mathbb{N}} \rightarrow \mathbb{R}$ with

$$\tilde{Y}^{CY} = f(N^a, Y_1^a, Y_2^a, \dots, N^l, Y_1^l, Y_2^l, \dots).$$

The following non-exhaustive list of basic operations allows the definition of such a transformation function:

Name of the transformation	Parameters, symbols	Definition
Excess of loss	$f_{XoL}(d,l)\colon \mathbb{R} \to \mathbb{R}$	$f_{XoL}(d,l)(x) = \min\left(\max(x-d,0), l\right)$
Excess of loss to n segments* with the same conditions	$f_{XoL,n}(d,l)\colon \mathbb{R}^n \to \mathbb{R}^n$	$f_{XoL,n}(d,l)(x) = \left(f_{XoL}(d,l)(x_1), \dots, f_{XoL}(d,l)(x_n)\right)$
Excess of loss to n segments* with distinct conditions	$f_{XoL,n}(\vec{d},\vec{l})\colon \mathbb{R}^n \to \mathbb{R}^n$	$f_{XoL,n}(\vec{d},\vec{l})(x) = \left(f_{XoL}(d_1,l_1)(x_1), \dots, f_{XoL}(d_n,l_n)(x_n)\right)$
Trivial embedding	$\pi_1\colon \mathbb{R}^{n_1} \to \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ and $\pi_2\colon \mathbb{R}^{n_2} \to \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$	$\pi_1(x) = (x,0),\ \pi_2(y) = (0,y)$ $(\pi_1,\pi_2)(x,y) = \pi_1(x) + \pi_2(y)$
Trivial aggregate from segments*	$f_{sum}\colon \mathbb{R}^n \to \mathbb{R}$	$f_{sum}(x) = x_1 + \dots + x_n$
Quota share	$f_{QS}(\rho)\colon \mathbb{R} \to \mathbb{R}$	$f_{QS}(\rho)(x) = \rho \cdot x$
* The word "segment" is used generically here, e.g. a segment of an aggregate claim is a single claim.		

The captive model proposes a default transformation defined as follows.

For the underlying parameter segment m, five inputs can be provided by the captive, derived from its assumed reinsurance and ceded retrocession conditions:

- Annual aggregate limit $AAL \in (0,\infty]$
- Each and every loss limit $EEL \in (0,\infty]$
- Annual aggregate deductible $AAD \in [0,\infty)$
- Each and every loss deductible $EED \in [0,\infty)$
- Quota share $\rho \ \in (0,1]$

By default, $AAL = \infty,\ EEL = \infty,\ AAD = 0,\ EED = 0$ and $\rho = 1$ when the corresponding input is not provided. The default transformation into the captive claim $\tilde{y} = \tilde{y}_m$ of the underlying parameter segment m and the default aggregation of the parameter segments are defined by

Default transformation into captive claim
Default transformation into captive claim $\tilde{y} = \tilde{y}_m$ of the underlying parameter segment m
$\begin{array}{ll} y_1^a, \dots y_{n_a}^a \rightarrow f_{XoL,n_a}(EED, EEL) \searrow & \\ & (\pi_a, \pi_l) \rightarrow f_{sum} \rightarrow f_{XoL}(AAD, AAL) \rightarrow f_{QS}(\rho) \rightarrow \tilde{y} \\ y_1^l, \dots y_{n_l}^l \rightarrow f_{XoL,n_l}(EED, EEL) \nearrow & \end{array}$
Default transformation from CY parameter segments claims $\tilde{y}_m$ into the captive CY aggregate claim $\tilde{y}^{CY,t_0}$
$\tilde{y}_m$, $m \in$ CY-seg [m independent segments] $\rightarrow f_{sum} \rightarrow \tilde{y}^{CY,t_0}$

Please note that this default transformation may not be applicable; in particular, the following conditions apply:

- Reinsurance conditions across several LoBs cannot be calculated.
- The effect of reinstatement premiums is neglected (in case these are considerable, they should increase the net loss).
- The effect of sliding scale commissions and profit commissions have been neglected.
- The default transformation assumes that the reinsurance contracts are applied in the order given above.
- Applying both assumed reinsurance conditions and ceded retro conditions might not be possible.
- EEL and EED conditions cannot be applied meaningfully for aggregate modelling of parameter segments with expected frequency greater than 10, see also "Ground-up model" above.
- The default transformation concern plain traditional reinsurance structures without loss sensitive features. Loss sensitive conditions in reinsurance structures, like paid reinstatements, swing rates or sliding scale conditions, loss corridors or multi-year-structures with material impact on recoveries need also to be taken into account.

Companies can use the default transformation to determine the premium risk only if they have convinced themselves that none of the simplifications above or similar lead to a material underestimation of the risk.

Within the captive model, a company-defined transformation from ground-up claims modelled with the ground-up model above into the net captive claim (after retrocession) is permitted. In that case, the captive describes the selected transformation.

If there are dependencies between CY parameters segments such that the independence assumption between CY parameters segments leads to a material underestimation of solvency, the captive has to apply for a company-specific adjustment in the sense of mn 107-109 FINMA Circ. 17/3 "SST", unless these dependencies are sufficiently modelled by *Maximal possible loss modelling* (see below) or by an appropriate IE3 event (see Section 7).

The ground-up modelling has provided a distribution of $\tilde{Y}^{CY,t_0}$, allowing to obtain that of

$$X^{CY} \approx d^{CY} \cdot \left(\tilde{Y}^{CY,t_0} - BE_{t_0}^{(N)}\left(\tilde{Y}^{CY,t_0}\right)\right)$$

where

$$d^{CY} = \frac{BE_{t_0}\left(\tilde{Y}^{CY,t_0}\right)}{BE_{t_0}^{(N)}\left(\tilde{Y}^{CY,t_0}\right)}$$

6.3 Maximal possible loss modelling

The captive can select CY parameter segments where instead of ground-up modelling the following conservative approach is used.

For such a parameter segment m with a maximal possible net loss to the captive MPL_m and expected net loss EL_m, it is assumed that the loss amount MPL_m is almost surely an upper bound to $\tilde{Y}_m^{CY,t_0}$. As a simplification, no discounting effect is taken into account.

The random variable X_m^{CY} in focus is replaced by a deterministic upper bound, that is:

$$X_m^{CY} \approx MPL_m - EL_m$$

The captive provides values for MPL_m and EL_m with explanations. It is possible that some segments may be modelled with the MPL approach and other segments may be modelled with the ground-up approach.

7 Individual events risk (IE3)

Each captive is subject to specific risk stemming from the economic activities of the group owning the captive. Part of this risk is (to a large extend) not covered by estimations from data history and/or by the standard modelling of insurance risk in the captive model.

Therefore the captive company has to define one to three scenarios related to its own exposure and evaluate these by their occurrence probability and impact related to insurance risk. These scenarios will be aggregated WITHIN the insurance risk (see Section 8) and are part of the captive model, under the name *individual events* (IE3; "3" is to differentiate them from the IE1 and IE2 scenarios used in StandRe).

IE3 scenarios can be defined as a classical event (e.g. an industrial accident), but may also be understood in a broader sense, e.g. that a limit is reached (whatever is the underlying event) or that a court decision has an impact on the company insurance business. IE3 scenarios typically cover events with low occurrence probability and severe impact, but may also cover events with larger occurrence probability. The trigger is that IE3 risk covers insurance risk that is not (enough) represented by the premium risk model or the reserve risk model, e.g. it has not yet been observed in the data which were used for the calibration of the model parameters.

The overarching scenarios of the SST (as described in the *Technische Beschreibung Szenarien / Description technique scénarios)* remain outside the scope of the captive model. Their aggregation, respectively non aggregation, into target capital is explained there, most notably concerning concentrations. An IE3 scenario modelled within the insurance risk should not be aggregated again as a scenario to the overall one-year risk.

The distribution of three IE3 scenarios S_1, S_2 and S_3 is discrete and specified by the probabilities and impacts of the seven combinations. For the case of three IE3 scenarios, at least the occurrence probabilities p_1, p_2 and p_3 as well as the impacts $\check{c}_1, \check{c}_2$ and $\check{c}_3$ (with loss as positive value) when no other scenario occurs have to be given by the captive. Default formulas are given below based on the assumption of independent scenarios and without special reinsurance conditions, but the input can be changed by the captive. In particular, if p_1, p_2 and p_3 are small enough, one can assume that at most one scenario occurs, by setting $w_1 = p_1$, $w_2 = p_2$, $w_3 = p_3$ and the combined w_is to nil.

Name of the IE3 scenario combination	IE3 scenario combination	occurrence probability of the combination	impact of the combination
S_1 only	$S_1 \setminus (S_2 \cup S_3)$	$w_1 = p_1 \cdot (1-p_2) \cdot (1-p_3)$	$\check{c}_1 = -c_1$
S_2 only	$S_2 \setminus (S_1 \cup S_3)$	$w_2 = (1-p_2) \cdot p_2 \cdot (1-p_3)$	$\check{c}_2 = -c_2$
S_3 only	$S_3 \setminus (S_1 \cup S_2)$	$w_3 = (1-p_1) \cdot (1-p_2) \cdot p_3$	$\check{c}_3 = -c_3$
S_1 and S_2 only	$(S_1 \cap S_2) \setminus S_3$	$w_{1,2} = p_1 \cdot p_2 \cdot (1-p_3)$	$\check{c}_{1,2} = \check{c}_1 + \check{c}_2$
S_1 and S_3 only	$(S_1 \cap S_3) \setminus S_2$	$w_{1,3} = p_1 \cdot (1-p_2) \cdot p_3$	$\check{c}_{1,3} = \check{c}_1 + \check{c}_3$
S_2 and S_3 only	$(S_2 \cap S_3) \setminus S_1$	$w_{2,3} = (1-p_1) \cdot p_2 \cdot p_3$	$\check{c}_{2,3} = \check{c}_2 + \check{c}_3$
S_1 and S_2 and S_3	$S_1 \cap S_2 \cap S_3$	$w_{1,2,3} = p_1 \cdot p_2 \cdot p_3$	$\check{c}_{1,2,3} = \check{c}_1 + \check{c}_2 + \check{c}_3$

The individual events IE3 are represented by a discrete random variable X^{IE3} and a distribution, defined by eight values and probabilities, the seven above and $\check{c}_0 = 0$ with w_0, the probability that no IE3 events occurs, given by $w_0 + w_1 + w_2 + w_3 + w_{1,2} + w_{1,3} + w_{2,3} + w_{1,2,3} = 1$.

8 Aggregation into the non-life insurance risk

From the previous sections we have the decomposition

$$Z^{NL\text{-}insurance\text{-}risk} = -X^{PY} - X^{CY} - \epsilon^{PY} - \epsilon^{CY} - \epsilon^{URR}$$

where $\epsilon^{PY}, \epsilon^{CY}$ and ϵ^{URR} represent the risks not modelled by the reserve risk model (see Section 5) or by the premium risk model (see Section 6), respectively.

The individual events enhance the reserve risk model and premium risk model by assuming the relation:

$$\epsilon^{PY} + \epsilon^{CY} + \epsilon^{URR} = X^{IE3} + \epsilon^{NL\text{-}insurance\text{-}risk}$$

i.e.

$$Z^{NL\text{-}insurance\text{-}risk} = -X^{PY} - X^{CY} - X^{IE3} - \epsilon^{NL\text{-}insurance\text{-}risk}$$

where $\epsilon^{NL\text{-}insurance\text{-}risk}$ is an independent random variable for the remaining error term, neglected in the captive model.

In the special case where all CY parameter segments are modelled with the MPL approach (see Section 6.3) for X^{CY}, one can neglect X^{IE3} in the above relation if it can be justified that the risk of X^{IE3} is sufficiently covered.

The previous sections provided a distribution to each of the random variables X^{PY}, X^{CY} and X^{IE3}. For the required distribution of the sum $-X^{PY} - X^{CY} - X^{IE3}$, the captive model assumes:

- X^{PY} and X^{CY} are comonotone;
- X^{IE3} is independent of $X^{PY} + X^{CY}$.

Writing F_0 for the cumulative probability distribution of $-X^{PY} - X^{CY}$, the convolution with the discrete distribution of $-X^{IE3}$ yields the cumulative probability distribution F of $-X^{PY} - X^{CY} - X^{IE3}$:

$$\begin{aligned} F(z) = w_0 \cdot F_0(z) + w_1 \cdot F_0(z - c_1) + w_2 \cdot F_0(z - c_2) + w_3 \cdot F_0(z - c_3) \\ + w_{1,2} \cdot F_0\left(z - c_{1,2}\right) + w_{1,3} \cdot F_0\left(z - c_{2,3}\right) + w_2 \cdot F_0\left(z - c_{2,3}\right) + w_{1,2,3} \cdot F_0\left(z - c_{1,2,3}\right) \end{aligned}$$

where the ws and the $c = -\check{c}$s are given by the individual events IE3 specification of Section 7.

In the particular case where it is assumed that IE3 scenarios are pairwise excluding (at most one scenario can occur in the year), the second line is nil. This is the SST standard aggregation method for scenarios albeit at the insurance risk distribution level, see *Technische Beschreibung für das SST-Standardmodell Aggregation und Mindestbetrag / Description technique du modèle standard SST pour l'agrégation et le montant minimum.*

In the general case, setting S_1', S_2', S_3', S_4', S_5', S_6' and S_7' as new scenarios defined by each mutually exclusive combination of S_1, S_2 and S_3, the SST standard aggregation method applied to the S's provides the above formula. This expansion of the aggregation method is especially useful if an IE3 scenario is set with relatively high probability, meaning that the underlying assumption of at most one scenario occurring within one year is likely to be violated.

The stand-alone non-life insurance risk provided by the captive model distribution F is given by

$$-ES_{\alpha}(Z^{NL\text{-}insurance\text{-}risk}) = ES^{1-\alpha}(X^{PY} + X^{CY} + X^{IE3})$$

where the expected shortfall ES_{α} is defined on the left side of a profit loss random variable, see margin no 58 of FINMA Circ. 17/3 "SST", and $ES^{1-\alpha}(S) = -ES_{\alpha}(-S)$ was defined at Section 4.2 for a random variable S. Moreover, by comonotonicity assumption

$$ES^{1-\alpha}(X^{PY} + X^{CY}) = ES^{1-\alpha}(X^{PY}) + ES^{1-\alpha}(X^{CY})$$

9 Expected insurance result of new business

Recall that from the one-year change in scope (see Section 4.2), the expected insurance result was defined as

- expected non-life insurance result = net expected premiums minus net expected discounted losses minus expected expenses of the new business,

reflecting only the business which is not in the balance sheet at $t = 0$. Here a gain is represented by a positive value.

In line with art. 30 para. 1 ISO, the expected non-life insurance result is required to include all costs (including administrative and overhead costs) needed for the own fulfilment of the insurance liabilities written until time t_1 under the assumptions of art. 2 para. 2 and 3 ISO-FINMA ("run-off").

The captive has to estimate its expected insurance result, typically by using budget figures (cf. art. 2 para. 1 ISO-FINMA). As a simplification the expected value stemming from individual events can be set to nil.

The error due to the estimation of the expected non-life insurance result is neglected.

In case of gross instead of net (of ceded retrocession) modelling of the reserve risk and/or premium risk, the expected insurance result has to be computed on a net basis nonetheless. This is necessary for consistency with the SST balance sheet.

The stand-alone one-year risk capital due to non-life insurance business is given by the stand-alone non-life insurance risk minus the expected non-life insurance result.

10 Input from the captive model into other modules

This section describes the necessary inputs from the insurance business that are required for the calculation of the market risk and that are necessary for the aggregation with the other risk categories, such as market risk and credit risk. This section also explains the computation of the market value margin (MVM) from the inputs of the captive model.

10.1 Cash flows from insurance business

In order to calculate the market risk and its components like FX and spread risk, the cashflows from the insurance business are a necessary input.

This input is derived as follows: The positions of the SST balance sheet as described in Section 3.1 are grouped to produce net figures. Payment patterns of the captive model are then used to produce the suitable cash flows. Alternatively the company can input its own cash flows if justified. This is the same approach as in StandRe, cf. the detailed explanations given in the technical description for the SST standard model reinsurance (StandRe), Section 8.2.

10.2 Market value margin (MVM)

According to the *technical description for the SST standard model aggregation and market value margin*, the market value margin at time t_0 is a sum of components of which $MVM_{Captive}$ is defined in the current document.

In the following, the concrete computation of $MVM_{Captive}$ is documented. As a simplifying assumption, the captive model assumes that the non-hedgeable market risk, the credit risk of ceded retrocession and possible scenarios are neglected in the calculation of the MVM as explained below. Alternatively the captive can compute its MVM by the method of StandRe and follow the assumptions of that method, see also the technical description for the SST standard model reinsurance (StandRe), Section 8.3.

Consistent to the *technical description for the SST standard model aggregation and market value margin*, it is assumed that

$$MVM_{Captive} = \eta_{CoC} \cdot \sum_{k \geq 1} v_{k+1} \cdot SCR_{t_k}$$

where η_{CoC} is the cost of capital rate prescribed by FINMA and SCR_{t_k} is the expected one-year risk capital at t_k, i.e. for the period t_k to t_{k+1}, due to captive insurance risks. The following explains the method and assumptions behind the computation of SCR_{t_k} for $k \geq 1$.

For this we define a convenient partition of the cash flows outstanding at time t_k for $k \geq 1$ regarding the business in scope (see Sections 4.3 and 4.4) as follows:

<table>
<tr><th colspan="5">Partition of business in scope of SST along existing business at t_0 (i.e. incepted until t_0)
and new business between t_0 and t_1</th></tr>
<tr><th colspan="3">regarding time t_0, for one-year risk from t_0 to t_1</th><th colspan="2">regarding time $t_k \geq t_1$, for one-year risk from t_k to t_{k+1}</th></tr>
<tr><td>Reserve risk</td><td>PY</td><td>existing at $t = 0$ and earned at $t = 0$</td><td>existing at $t = 0$ and earned at $t = 0$</td><td rowspan="5">PY_k</td></tr>
<tr><td rowspan="7">Premium risk</td><td rowspan="2">CY</td><td>existing at $t = 0$ and unearned at $t = 0$ but earned at $t = 1$</td><td>existing at $t = 0$ and unearned at $t = 0$ but earned at $t = 1$</td></tr>
<tr><td>new between $t = 0$ and $t = 1$ and earned at $t = 1$</td><td>new between $t = 0$ and $t = 1$ and earned at $t = 1$</td></tr>
<tr><td rowspan="5">URR</td><td rowspan="5">existing at $t = 0$ and unearned at $t = 1$

new between $t = 0$ and $t = 1$ and unearned at $t = 1$</td><td>existing at $t = 0$ and unearned at $t = 1$ but earned at $t = k$</td></tr>
<tr><td>new between $t = 0$ and $t = 1$ and unearned at $t = 1$ but earned at $t = k$</td></tr>
<tr><td>existing at $t = 0$ and unearned at $t = k$ but earned at $t = k + 1$</td><td rowspan="2">CY_k</td></tr>
<tr><td>new between $t = 0$ and $t = 1$ and unearned at $t = k$ but earned at $t = k + 1$</td></tr>
<tr><td>existing at $t = 0$ and unearned at $t = k + 1$</td><td>URR_k</td></tr>
</table>

As a simplifying assumption, the captive model neglects the hatched boxes of this partition referring to the URR. It means for $k \geq 1$ that $SCR_{t_k} = SCR_{t_k}^{PY_k}$, i.e. the CY_k risk and URR_k risk are neglegted, and that PY_k risk comes only from PY and CY.

Additional simplifying assumptions are $SCR_{t_k} = SCR_{t_k}^{PY} + SCR_{t_k}^{CY}$ (i.e. PY and CY are comonotone), and further partitioning in PY parameter segments and CY parameter segments it is assumed that:

$$SCR_{t_k} = \sum_{\ell \in PY\text{-}seg} SCR_{t_k}^{PY,\ell} + \sum_{\ell \in CY\text{-}seg} SCR_{t_k}^{CY,\ell}$$

The following expressions of the expectations by segment ℓ for cash flows outstanding at t_k are applied later on:

$$E\left(BE_{t_k}^{(N),PY,\ell,(t_k)}\right) = BE_{t_0}^{(N),PY,\ell} \cdot \sum\nolimits_{j \geq k+1} \pi_j^{PY,\ell}, \quad E\left(BE_{t_k}^{PY,\ell,(t_k)}\right) = BE_{t_0}^{(N),PY,\ell} \cdot \sum\nolimits_{j \geq k+1} \nu_j \cdot \pi_j^{PY,\ell}$$

$$E\left(BE_{t_k}^{(N),CY,\ell,(t_k)}\right) = BE_{t_0}^{(N),CY,\ell} \cdot \sum_{j\geq k+1} \pi_j^{CY,\ell}, \quad E\left(BE_{t_k}^{CY,\ell,(t_k)}\right) = BE_{t_0}^{(N),CY,\ell} \cdot \sum_{j\geq k+1} \nu_j \cdot \pi_j^{CY,\ell}$$

$BE_{t_0}^{(N),PY,\ell}$, the best estimate of reserves for PY business at time t_0 and the corresponding incremental expected payment pattern $\pi_j^{PY,\ell}$ are given by the reserve risk model. $BE_{t_0}^{(N),CY,\ell}$, the expected losses for new business in the current year and the corresponding payment pattern $\pi_j^{CY,\ell}$ are given by the premium risk model. Related discount factors to time t_0 are calculated from the inputs as follows $d_\ell^{PY} = \sum_{j\geq 1} \nu_j \cdot \pi_j^{PY,\ell}$ and $d_\ell^{CY} = \sum_{j\geq 1} \nu_j \cdot \pi_j^{CY,\ell}$.

We can now provide a formula computing $SCR_{t_k}^{PY,\ell}$ (and similarly $SCR_{t_k}^{CY,\ell}$) for each PY parameter segment ℓ. The captive model assumes:

$$\nu_k \cdot SCR_{t_k}^{PY,\ell} = \delta_k^{PY,\ell} \cdot SCR_*^{PY,\ell}$$

where

$$\delta_k^{PY,\ell} = \frac{E\left(BE_{t_k}^{PY,\ell,(t_k)}\right)}{BE_{t_0}^{PY,\ell}} = \frac{\sum_{j\geq k+1} \nu_j \cdot \pi_j^{PY,\ell}}{\sum_{j\geq 1} \nu_j \cdot \pi_j^{PY,\ell}} = 1 - \frac{\sum_{j=1}^{k} \nu_j \cdot \pi_j^{PY,\ell}}{d_\ell^{PY}}$$

is a run-off factor based on discounted best estimates and $SCR_*^{PY,\ell}$ is an auxiliary one-year risk capital at t_0 as explained further below.

Writing $\mu_\ell^{PY} = BE_{t_0}^{PY,\ell}$ and CV_ℓ^{PY} for the mean and coefficient of variation selected in the PY parameter segment ℓ of the reserve risk model, we assume that

$$SCR_*^{PY,\ell} = (f_{1-\alpha}(CV_\ell^{PY}) - 1) \cdot \mu_\ell^{PY} \cdot d_\ell^{PY}$$

where, writing Φ for the cumulative standard normal distribution, the factor $f_{1-\alpha}(CV)$ for the risk measure expected shortfall at confidence level $1-\alpha$ and given a coefficient of variation *CV* is defined by:

$$f_{1-\alpha}(CV) = \frac{1}{\alpha} \cdot \left(1 - \Phi\left(\Phi^{-1}(1-\alpha) - \sqrt{\log(1+CV^2)}\right)\right)$$

Remark 1: μ_ℓ^{PY} can be reduced by the net reserve amount which is not under risk, for claims that are reserved in the SST-balance sheet to the full contract limit without further deterioration possible.

Remark 2: If $X = d \cdot (Y - E(Y))$ with Y lognormally distributed and coefficient of variation CV_Y and with $d > 0$ (here d is a discount factor), the following property holds:

$$ES^{1-\alpha}(X) = d \cdot (f_{1-\alpha}(CV_Y) - 1) \cdot E(Y);$$

for the derivation, see formula (145) of the technical description for the SST standard model non life.

It follows that the formula for the discounted MVM used in the captive model is given by:

$$MVM_{Captive} = \eta_{CoC} \cdot \sum_{k \geq 1} v_{k+1} \cdot \left(\sum_{\ell \in PY\text{-}seg} SCR_{t_k}^{PY,\ell} + \sum_{\ell \in CY\text{-}seg} SCR_{t_k}^{CY,\ell} \right)$$
$$\approx \eta_{CoC} \cdot \sum_{k \geq 1} \left(\sum_{\ell \in PY\text{-}seg} v_k^{Curr_\ell} \cdot SCR_{t_k}^{PY,\ell} + \sum_{\ell \in CY\text{-}seg} v_k^{Curr_\ell} \cdot SCR_{t_k}^{CY,\ell} \right)$$

where for $\ell \in PY\text{-}seg$

$$v_k^{Curr_\ell} \cdot SCR_{t_k}^{PY,\ell} = \mu_\ell^{PY} \cdot \left(f_{1-\alpha}\left(CV_\ell^{PY}\right) - 1\right) \cdot \sum_{j \geq k+1} v_j^{Curr_\ell} \cdot \pi_j^{PY,\ell}$$

and for $\ell \in CY\text{-}seg$

$$v_k^{Curr_\ell} \cdot SCR_{t_k}^{CY,\ell} = \mu_\ell^{CY} \cdot \left(f_{1-\alpha}\left(CV_\ell^{CY}\right) - 1\right) \cdot \sum_{j \geq k+1} v_j^{Curr_\ell} \cdot \pi_j^{CY,\ell}$$

$Curr_\ell$ denotes the currency of the underlying parameter segment ℓ and the second line of the formula is implemented in the captive model.

11 IT implementation

11.1 Description of the *SST-Captive-Template.xlsx*

The Excel workbook *SST-Captive-Template* is intended to collect all parameters and to support some computations related to the captive model. We explain the purpose and possible special feature of each worksheet below.

All values have to be entered in millions of SST currency throughout the template.

11.1.1 Intro_SM_Captive

This sheet states the purpose of the template and asks for a few company specific general inputs used throughout the template.

11.1.2 CA_update

Contains the change log of the template.

11.1.3 CA_prescribed_parameters

This sheet contains the FINMA prescribed parameters, such as yield curves and FX exchange rates. Parameters relating to 31 December are updated in January. In case JPY is used, please copy the yield curve from the *SST-StandRe-Template* here.

11.1.4 CA_calculation_documentation

This sheet is intended to explain how the captive model was specifically applied by the company.

A selection of topics is listed (cf. columns "Field" and "Description/question"). Computations from different sheets are provided as information and helper for consistency checks (cf. column "Value").

Where explanations are needed for a third person familiar with the topic to understand the approach, please provide comments (art. 24 para. 3 let. e ISO-FINMA). In particular, this applies to places where the model allows for company-specific inputs or alternative approaches.

Explanations can be provided directly in this sheet (in the column "Comments/explanations"), or a precise reference to the SST report (e.g. page, section) can be given instead.

11.1.5 CA_reserves_and_premiums

This is a reporting sheet to provide a standardised overview of the captive portfolio. It is based on a similar sheet already used in StandRe. The data must be entered by the company from column N onwards. The information in this sheet is not directly linked to the risk calculation.

11.1.6 CA_reserve_risk

This sheet reports the parameters of the reserve risk model. The reserve risk parameters have to be entered by the company per parameter segment.

Undiscounted reserves, column *alternative*: If a full limit loss has been reached for a segment, there is likely no more reserve risk: the reserve in the balance sheet is considered as deterministic. In that case, it is possible to write the value 0 instead of the effective reserve value.

Coefficient of variation, column *alternative*. If the company uses its own coefficients of variation, they have to be entered here.

11.1.7 CA_premium_risk

This sheet reports the parameters of the premium risk model. The premium risk parameters have to be entered by parameter segment by the company.

For parameter segments with a frequency of attritional claims greater than 10, the transformation of parameters for an aggregate loss model instead of a frequency-severity model is implemented in the sheet *CA_input_SST_Template.*

11.1.8 CA_expected_result

In this sheet, the company needs to enter all the relevant information for the calculation of the expected insurance result. The expected insurance result of the new business, i.e. incepting between $t = 0$ and

$t = 1$, is computed. An additional input for the calculation of the market value margin (MVM) is also entered in this sheet.

11.1.9 CA_IE3_risk

The company must propose one to three business-specific scenarios for insurance risks that have not been modelled (such as previously unobserved or emerging risks) or have been insufficiently modelled in the reserve risk or premium risk models. Occurrence probabilities and impacts are specific to the captive company and have to be explained. More than one of these so-called individual event (IE3) scenarios can occur within one year (this should typically be the case if an occurrence probability is "high", e.g. 10%), and the aggregation is within insurance risk, i.e. with reserve risk and premium risk.

11.1.10 CA_discount_factors

In this sheet, the payment patterns by parameter segment have to be entered. This sheet provides the information needed in other sheets using pattern information, and in particular computes the PY discount factor for reserve risk and the CY discount factor for premium risk.

11.1.11 CA_MVM

Computes the MVM. The necessary inputs are linked from the other sheets. There is no additional input required by the company.

11.1.12 CA_insurance_cash_flows

In this sheet, the cash flows stemming from the insurance business of the captive are calculated for the market risk model.

The company has to enter the positions of the SST balance sheet (to be found in the *SST-Template*) which are related to the insurance business. The breakdown into currency and into cash flows is linked from inputs of the other sheets used for the captive model. The company may provide alternatives to these default-calculated cash flows, with explanations.

11.1.13 CA_input_SST_Template

This sheet collects inputs from other sheets which are required by the *SST-Template* from the captive model and which have to be copied into the *SST-Template*.

Each section of this sheet corresponds to a specific sheet of the *SST-Template*. Further explanations can be found, in particular regarding the aggregation of reserve risk, premium risk and IE3 risk into insurance risk, and regarding the decomposition of insurance risk into PY and CY as shown in the FDS.

If all CY parameter segments have been modelled with the MPL approach and the IE3 risk can be neglected, see Section 8, it is permitted to change the probabilities of the IE3 scenarios to nil in that sheet.

As the non-hedgeable market risk is neglected for captives, see *Technical Description Aggregation and Market Value Margin*, the value $\chi_{Captive}$ is set to nil – and $\widetilde{BE}_{Captive} \coloneqq BE_{Captive}$ has no relevance.

11.2 Features in *R-tool*

For the implementation of the *R-Tool* please consult the section *System requirements of the executable version* in the document *IT Notes*.

The captive specific features in the *R-Tool* for the calculation of the insurance risk include the following

- Modelling of CY risk by CY parameter segment (with indicator for the CY parameter segment model: ground-up loss or maximal possible loss, see Section 6)
  - by CY parameter segment modelled by ground-up loss approach according to the specification in the template:
    - As frequency severity model for attritional claims with frequency (Poisson), mean and standard deviation for the gamma distribution as inputs
    - As frequency severity model for large claims with frequency (Poisson), shape and threshold (scale) for the Pareto distribution as inputs
    - Transformation into a yearly loss distribution CY parameter segment according to the standard transformation defined at Section 6.2.2.
  - by CY parameter segment of maximal possible loss model as a deterministic distribution with maximum net loss and expected net loss as input
- Aggregation of the net losses across the parameter segments for CY independently, see Section 6
- Aggregation of PY risk and CY risk comonotonic, before IE3 risk, see Section 8

To run the above features with the *R-Tool,* select « captive » in the sheet *Non Life* of the *SST-Template.* Else, in particular if the standard transformation cannot be used, select « simulations » or « cumulative distribution function », and enter the corresponding simulations or cumulative distribution function obtained by your external tool in this sheet.

- The *R-Tool* aggregates the individual events IE3 given as input in the sheet *Non Life* of the *SST-Template,* regardless of whether « captive », « simulations » or « cumulative distribution function » has been chosen, see Section 8.
- The FDS shows the stand-alone non-life insurance risk with and without IE3 risk. If « captive » has been selected, the PY risk and CY risk shown in the FDS is computed by the *R-Tool* from the parameters of the captive model.

12 Appendix

It may happen that the company's segmentation built and used for internal purposes does not match the specific segmentation for the capital risk model, i.e. the SST standard model for reinsurance captive. In this case, the properties of Section 12.1 on moments should be used. Section 12.2 provides the most common properties when working with lognormal distributions. In particular, relationships are shown when lognormal distributions are assumed at the segment level or on an aggregate segment. Section 12.3 shows the parametrisation used here for the Gamma distribution.

12.1 Moment aggregation and disaggregation

Let $X = X_1 + \cdots + X_n$ be a decomposition into n segments of a random variable X with positive values. Denote the corresponding mean and standard deviation by $\mu = E[X]$, $\sigma = \sqrt{Var(X)}$, $\mu_i = E[X_i]$ and $\sigma_i = \sqrt{Var(X_i)}$, and let Γ_{ij} for $i, j = 1, \dots, n$ be the components of the corresponding correlation matrix Γ. The related coefficients of variation are $CV = \mu/\sigma$ and $CV_i = \mu_i/\sigma_i$.

12.1.1 Moment aggregation

From mean and standard deviation per segment, aggregated mean and aggregated standard deviation are derived by:

$$\mu = \sum_{i=1}^{n} \mu_i \qquad \sigma = \sqrt{\sum_{i,j} \sigma_i \cdot \sigma_j \cdot \Gamma_{ij}} = \sqrt{\vec{\sigma}^T \, \Gamma \, \vec{\sigma}}$$

If $\Gamma_{ij} = \rho$ for $i, j = 1, \dots, n$ and $i \neq j$, i.e. the same correlation $\rho > 0$ is assumed between distinct pairs, one has

$$\sigma = \sqrt{\rho \cdot \sum_{i \neq j} \sigma_i \cdot \sigma_j + \sum_i \sigma_i^2} = \sqrt{\rho \cdot \sum_{i,j} \sigma_i \cdot \sigma_j + (1-\rho) \cdot \sum_i \sigma_i^2} = \sqrt{\rho \cdot \left(\sum_i \sigma_i\right)^2 + (1-\rho) \cdot \sum_i \sigma_i^2}$$

12.1.2 Moment disaggregation

Assume we have an aggregated coefficient of variation CV and want to get the coefficients of variation of the segments CV_k. Based on the assumption that all segments have the same coefficient of variation $CV_k = CV_1$, the disaggregated standard deviation σ_k of a segment k is given by

$$\sigma_k = \sigma \cdot \frac{\mu_k}{\sqrt{\sum_{i,j} \mu_i \cdot \mu_j \cdot \Gamma_{ij}}} \quad \text{or equivalently} \quad CV_k = CV \cdot \frac{\sum_{i=1}^{n} \mu_i}{\sqrt{\sum_{i,j} \mu_i \cdot \mu_j \cdot \Gamma_{ij}}}$$

and the coefficients of variation CV_k of the segments are greater than the aggregated coefficient of variation CV unless all correlations Γ_{ij} are equal to one. In that case, $CV_k = CV$.

Proof. The formula follows from

$$\sigma = \sqrt{\sum_{i,j} \sigma_i \cdot \sigma_j \cdot \Gamma_{ij}} = CV_1 \cdot \sqrt{\sum_{i,j} \frac{\sigma_i}{CV_i} \cdot \frac{\sigma_j}{CV_j} \cdot \Gamma_{ij}}$$

The inequality relation between CVs is due to

$$\sum_{i,j} \mu_i \cdot \mu_j \cdot \Gamma_{ij} = 2 \sum_{i<j} \mu_i \cdot \mu_j \cdot \Gamma_{ij} + \sum_i \mu_i^2 \cdot \Gamma_{ii} \leq 2 \sum_{i<j} \mu_i \cdot \mu_j + \sum_i \mu_i^2 = \left(\sum_i \mu_i\right)^2$$

with equality if and only if Γ_{ij} =1 for $i, j = 1, \dots n$ ■

12.2 Properties related to the lognormal distribution

12.2.1 Reminder of properties related to a lognormal random variable

Assume Y is a lognormal distributed random variable with parameters μ_{lnorm} and σ_{lnorm}, i.e. $\frac{(\log(Y)-\mu_{lnorm})}{\sigma_{lnorm}}$ is standard normal distributed.

Writing μ_Y, σ_Y and $CV_Y = \sigma_Y / \mu_Y$ for mean, standard deviation and coefficient of variation of Y, respectively, the parameters of the lognormal distribution are given by $\mu_{lnorm} = \log(\mu_Y) - 0.5 \cdot \sigma_{lnorm}^2$ and $\sigma_{lnorm} = \sqrt{\log(1 + CV_Y^2)}$. Conversely, $\mu_Y = \exp(\mu + 0.5 \cdot \sigma_{lnorm}^2)$ and $\sigma_Y = \mu_Y \cdot \sqrt{\exp((\sigma_{lnorm}^2) - 1}$. Moreover, if $Y' = d \cdot Y$ with $d > 0$, then Y' is lognormal distributed with parameters $\mu'_{lnorm} = \mu_{lnorm} + \log(d)$ and $\sigma'_{lnorm} = \sigma_{lnorm}$. This property is used for discounting, where $d > 0$ is a deterministic discount factor.

The u-quantile and the right-hand expected shortfall at confidence level $1 - \alpha$ are given respectively by:

$$q_u(Y) = \inf\{y | P(Y \leq y) \geq u\} = exp\big(\mu + \sigma \cdot \Phi^{-1}(u)\big)$$

$$ES^{1-\alpha}(Y) = \frac{1}{\alpha} \int_{1-\alpha}^{1} q_u(Y) du = f_{1-\alpha}(CV_Y) \cdot E(Y)$$

where $f_{1-\alpha}(c) = \frac{1}{\alpha} \cdot \left(1 - \Phi\left(\Phi^{-1}(1-\alpha) - \sqrt{\log(1 + c^2)}\right)\right)$ with $\Phi^{-1}(1-\alpha)$ the $1 - \alpha$ quantile and Φ the cumulative distribution function of a standard normal distribution.

When discounting with a deterministic discount factor $d > 0$ and centering, the right-hand expected shortfall is given by the formula

$$ES^{1-\alpha}\big[d \cdot \big(Y - E(Y)\big)\big] = d \cdot [f_{1-\alpha}(CV) - 1] \cdot E(Y)$$

12.2.2 Properties of a sum of lognormal random variables

Let $Y_1, \dots, Y_n$ be n lognormal distributed random variables with mean $\mu_k = E(Y_k)$, standard deviation $\sigma_k = sd(Y_k)$ and coefficient of variation $CV_k = \sigma_k / \mu_k$ for $k = 1, \dots, n$, and Γ_{kl} their correlations for $k, l = 1, \dots, n$.

$ES^{1-\alpha}(Y_1) + \cdots + ES^{1-\alpha}(Y_n) \geq ES^{1-\alpha}(Y_1 + \cdots + Y_n)$ and usually $Y_1 + \cdots + Y_n$ is not lognormal distributed.

If $CV_1 = CV_2 = \cdots = CV_n$ and $\Gamma_{kl} = 1$ for $k, l = 1, \dots, n$, then $ES^{1-\alpha}(Y_1) + \cdots + ES^{1-\alpha}(Y_n) = ES^{1-\alpha}(Y)$, where Y is a lognormal distributed random variable with mean $\mu = \sum_{k=1}^{n} \mu_k$, standard deviation $\sigma = \sum_{k,l} \sigma_k \cdot \sigma_l$, and coefficient of variation $CV = CV_1$.

Proof. From moment disaggregation above, $CV = CV_1$. With the expected shortfall formula for a lognormal distribution, $\sum_k ES^{1-\alpha}(Y_k) = \sum_k f_{1-\alpha}(CV_k) \cdot \mu_k = f_{1-\alpha}(CV) \cdot \mu = ES^{1-\alpha}(Y)$ ∎

12.3 Gamma distribution

The parameters are $k > 0$ for the shape and $\theta > 0$ for the scale. The probability density function on the support $x \in (0, \infty)$ is given by

$$f(x) = \frac{1}{\Gamma(k) \cdot \theta^k} \cdot x^{k-1} \cdot e^{-\frac{x}{\theta}}$$

and the parameters can be derived from mean m and standard deviation s by $k = \frac{m}{s}$ and $\theta = \frac{s^2}{m}$.

13 Record of changes

Where changes have been made they are marked in blue. Where a chapter has been fully revised only its title is marked in blue. Corrections of typos or presentation styles are not marked.

Version 31 October 2024, changes from 31 October 2023

- References and terminology are modified according to the modification of ISO-FINMA (AVO-FINMA/OS-FINMA) abrogating the FINMA Circular 17/3 "SST".
- Alignment of terminology with SST balance sheet and *technical description for the SST standard model aggregation and market value margin* in Section 3.1 and 10.2.

Version 31 October 2023, changes from 31 October 2022

- References and terminology are modified according to the modification of ISO (AVO/OS) in force the 1st January 2024. For general explanations according to the modified ISO, see the *technical description for the SST standard model aggregation and market value margin*. The captive model is unchanged.

Version 31 October 2022, changes from 31 October 2021

- A few further explanations are marked in dark blue, there is no model change
- Section 11.1.13: an explanation was added which relates to the input of the non-hegeable market risk

Version 31 October 2021

Changes compared to the version 31 October 2020 are as follows.

- Section *1 Aim*. The explanations have been rewritten.
- Section *2 Scope*: unchanged, up to a comment on company-specific adjustments.
- Section *3 Captives insurance business*: unchanged.
- Section *4 The one-year change in risk-bearing capital*:
  - Section 4.1: subscript m used for segments (formerly ℓ), else unchanged.
  - Section 4.2: sign error corrected in formula iv.
  - Section 4.3: formerly 5.1, else unchanged.
  - Section 4.4: formerly 5.2, else unchanged.
  - Section 4.5 *Currencies*: Section added. There are no changes compared to the shadow computation 2021.
- The model changes reflected in Sections 5 to 9 below are those tested in 2021, see *Manual for the shadow computation 2021 (SC 2021 captive).* There are no changes compared to the shadow computation, except the one displayed at Section 6.
  - Section *5 Reserve risk model* (formerly Section 6). This section has been entirely rewritten.
  - Section *6 Premium risk model* (formerly Section 7). This section has been entirely rewritten. There is an additional possibility for segmentation for URR compared to the shadow computation.
  - Section *7 Individual events riks (IE3).* This is a new section.
  - Section *8 Aggregation into the non-life insurance risk*. This section has been entirely rewritten.
  - Section *9 Expected insurance result of new business.* Section added.
- Section *10 Input from the captive model into the other modules* (formerly Section 9):
  - Section 9.1: unchanged.
  - Section 9.2: claims that are reserved in the SST balance sheet to the full contract limit without further deterioration possible can be ignored in the calculation of the MVM. This has been changed compared to the shadow computation 2021. Else unchanged.
- Section *11 IT Implementation* (formerly Section 10):

  - Section 11.1. The explanations regarding the *SST-Captive-Template* are copied from the *Manual for the shadow computation 2021 (SC 2021 captive).* The change compared to the shadow computation is that inputs must be provided in million SST currency, as it was the case for the official SST 2021. Changes in the *SST-Captive-Template* compared to the *SST-Captive-Template* provided for the shadow computation 2021 are directly described in the sheet *CA_Update*.
  - Section 11.2. The features of the *R-Tool* tested in the shadow computation 2021 continue to be used for the SST 2022. This section, compared to that of SST 2021, has been shortened for the information already in previous sections of this document and completed for the features not existing in the SST 2021. Change compared to the shadow computation 2021 is that the FDS shows also the stand-alone non-life insurance risk before aggregation of the IE3 risk.
- Former Section 11: removed.
- Section *12 Appendix*: added.
- Section *13 Record of changes to the captive model*: added.