

# SST Standard Model for Reinsurers (StandRe)

## Model background

5 March 2018



This document provides background on the StandRe model. It is provided for illustration purposes only and does not constitute an official StandRe document. The document may potentially be updated at any time.

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# 1 Model structure

## 1.1 Output granularity

The output of the StandRe components IE1 and IE2 in the default case is one frequency-severity model for the total info event loss with distributions that are fitted to a number of scenarios. Main reasons for this approach are:

- (1) *Simplicity*: The model is more trackable and its calibration less demanding. Only one frequency-severity model needs to be calibrated, and no dependencies between sub-models need to be considered. Per risk or per contract event conditions of outward retro do not need to be implemented in an IT application and only need to be considered in the calculations of the scenarios.
- (2) *Dependencies*: Dependencies between segments are implicitly captured through the scenario severities and do not need to be modeled through dependency assumptions/copulas. Reliance is made on extreme value theory (Generalized Pareto fitting) to account for combinations of segment losses not observed in the individual scenarios;
- (3) *Extrapolation/fitting*: The credibility of the frequency-severity model is considered to be higher if one model is fitted to total event losses from all scenarios instead of several frequency-severity models fitted to segment event losses.

Depending on the form of the outward retrocession, this approach needs to be adjusted for IE1 by using several IE1 model segments. However, the IE1 model for several IE1 model segments is still ultimately calibrated on the level of the sum of the severities over the IE1 model segments to the IE1 scenarios by comparing the two exceedance frequency curves.

## 2 Attritional events

### 2.1 Correlation matrices

The correlation matrices between LOBs for AER and AEP are different and are derived, respectively, from the SST non-life standard model by mapping the StandRe LOBs to the LOBs of the non-life standard model (see below). This ensures some consistency. The basic rationale for the correlation numbers is as follows:

- There is a base correlation of 0.15 present in any case as a lower bound;
- If two LOBs have a significant common risk driver, the correlation increases to 0.25;
- If they have two significant common risk drivers, the correlation is 0.5.

It is useful to consider that there are differences in causes of dependencies between AER and AEP:

- For AEP, correlations mainly arise from the exposure to common “external events”, e.g. damage events.
- For AER, correlations are additionally more strongly driven by “company-internal events”, specifically the best estimate reserving process.

In particular, correlations within AER may be present even when there is no reason to assume a correlation coming from “external events”.

The distinction between AER and AEP correlations is reflected in the correlation matrix between regions (and between the types of business Prop and NonProp):

- For AEP, the correlations are assumed to be relatively small as the defined regions are quite large and thus assumed to be little affected by common “external events”.
- For AER, on the other hand, “internal events” are more relevant. Thus, correlations are required not to be below the floor of 0.15.

The AER correlations between regions other than “not regional” are set to 0.5, as for example claims inflation might be geographically linked, e.g. through repair costs, e.g. linked to commodity prices. Note that the strength of this link likely depends on the LOB, with e.g. a stronger link for Marine or Aviation, even when these are not assigned to “not regional”.

For “not regional” contracts, the correlations to other regions are larger than they would be for contracts assigned to some region, as “external events” affect a “not regional” contract always in a region.

The selection of the correlations between Prop and NonProp is a compromise between the correlation between Prop and Non-Prop for the same LOB and region, where the realistic correlation may be lower than the selection, and for different LOBs and regions, where it may be higher than the selection.

The following mapping from SST non-life standard model LOBs to StandRe LOBs is used to set the StandRe correlations based on the non-life standard model. The correlations from the non-life standard model selected for StandRe are the largest correlations of corresponding non-life LOBs.

Non-life LOB	StandRE LOB
Motor Liability ( <i>MFH</i> ) Motor Hull ( <i>MFK</i> )	Motor
Property ( <i>Sach</i> ) <i>ES-Pool</i>	Property
General Liability ( <i>Haft</i> )	General liability
Accident mandatory ( <i>UVG</i> ) <i>UVG Renten</i> Accident optional ( <i>U.o. UVG</i> ) Workers Compensation ( <i>KoIK</i> ) Individual Health ( <i>Eink</i> )	Accident and health
Transport ( <i>Trans</i> ) Aviation ( <i>Luft</i> )	Marine, aviation and other transport (MAT)
Credit & Surety ( <i>F&amp;K</i> )	Financial losses
Legal ( <i>Rechts</i> ) Other ( <i>Andere</i> )	Other Non-life

## 3 Individual events

### 3.1 Classification of info events/non-experience scenarios

The following criteria, which are subsequently explained by examples, can be used for a classification of info events. This can in particular be useful for understanding the range of the aspects of the risk profile to be covered by non-experience scenarios and specifically also own scenarios. The classification criteria are:

- (A) Damaged insured risks (e.g. physical objects, persons, financial loss) and the degree to which they are localized (e.g. geographically)
- (B) Affected LOBs and potentially regions (e.g. standard or specialty LOBs)
- (C) Affected (number of) insurers and subset of the given reinsurer's cedants (e.g. one, few, many; e.g. 10 insurers are affected, of which 3 are cedants of the given reinsurer)
- (D) Information that becomes known in the current year about claims caused by the info event. This is expressed by the "phases of insured claims": claim that has (been) "caused", "occurred", "realized", "reported", "settled", and claim that is "expected" to manifest.
- (E) Type of "impact" of the info event. This follows from (D). (E.g. known new claims, expected new claims, changes in known or expected claims severities)
- (F) Contracts affected, specifically affected underwriting/accident years (current accident year, prior accident years). This follows from (D) and the coverage conditions on the inward reinsurance contracts (e.g. claims made, losses occurring).
- (G) Difference between ultimate and one-year risk (linked to (D), no difference if the information in the current year includes the ultimate loss)
- (H) Contract events that correspond to the info event
- (I) Severity of loss to affected programs relative to maximum possible loss for the program.

#### Example 1: typical damage event

As an example, a typical "damage event", which is a sudden catastrophic occurrence such as an explosion of a large industrial facility, can lead to

- (A) few large insured risks significantly damaged (e.g. building) and potentially many damaged insured risks (e.g. people injured or dead), both considerably geographically localized, which
- (B) typically affect a few LOBs, where
- (C) there might be several insurers affected by LOB due to coinsurance on large industrial risks.

- (D) known information in the current year typically includes “caused”, “occurred”, “realized”, “reported”, and maybe even “settled”, so that
- (E) the event results in known new claims on
- (F) the current underwriting year given the coverage conditions (e.g. losses occurring), and
- (G) there is possibly some difference between ultimate and one-year risk, but maybe not too much.
- For (H), the event is likely a contract event, so that
- (I) the severity is a fraction of the maximum possible contract event loss, maybe 100% at least for the few large insured risks.

As a result, in terms of the inward reinsurance portfolio of the reinsurer, the event corresponds to several, but not many large new claims losses to reinsurer on current year business, from not many programs for not many LOBs, with little difference between ultimate and one-year risk (because of a short duration between the phase “caused” and the phase “settled”). The event may correspond to one contract event for each of the involved LOBs.

An alternative damage event scenario of a “giant hail storm” has many similar characteristics, but with likely many more insured risks only partially damaged and geographically strictly localized but over a wider area. Due to the per event cover, this can lead to similar losses to reinsurer, with potentially more programs affected.

The two examples in particular correspond to a distinction that can be made for damage events between e.g.

- damage events that lead to large losses because a lower number of high value objects are heavily damaged (e.g. explosion of industrial facility), and
- “frequency events”, in which a large number of not particularly high value objects are somewhat damaged or many new claims arise, so that the large event loss is the result of accumulation (e.g. hail storm, legal change).

An argument can be made that “frequency events” can be more problematic as they can be more difficult to control.

### **Example 2: emerging liability mass tort**

As a different example, consider an emerging liability mass tort event (something like Asbestos). For such an info event,

- (A) very many insured risks can incur damages (e.g. people injured), likely only weakly localized, which are

- (B) typically covered in terms of LOBs by e.g. Product Liability (quite strongly localized) and through a spill over to Commercial Liability and Workers Compensation/Employers Liability (weakly localized), where
- (C) for Product Liability several and for Commercial Liability and Workers Compensation/Employers Liability very many insurers may be affected.
- (D) known information in the current year likely mainly contains “expected” and maybe some “reported”
- Leading for Commercial Liability and Workers Compensation/Employers Liability to
  - (E) many expected and maybe some reported new claims for
  - (F) typically the current but also potentially many prior years, especially when the cover is “losses occurring” or “action committed”, and
  - (G) with considerable difference between the one-year and the ultimate risk due to the large number of only “expected” new claims.
- For Product Liability, there may be
  - (E) several but not many “expected” or “reported” new claims potentially
  - (F) only for the current year if the corresponding cover is claims made, with some difference between ultimate and one-year risk.

## 3.2 Frequency calculation for experience scenarios incl. uncertainty

### Input data

The starting point is a collection of historical large info event losses  $x_j^k$  with occurrence year  $k = 1, \dots, n$  in the observation period of  $n$  years, where  $j = 1, \dots, m_k$  and  $k = n$  is the year prior to the current year  $CY = n + 1$ . The purpose of the following is to estimate from the historical large info event losses  $x_j^k$ :

- the expected number of losses for the current year
- the estimation uncertainty of this estimate

In StandRe, the above quantities are estimated for experience scenarios, which are a subset of the historical large info event losses defined in terms of their as-if adjusted severity. For simplicity, we treat scenarios and historical large info event losses as synonymous in this section.

Each loss  $x_j^k$  has an assigned reporting lag  $lag_j^k$  corresponding to the number of years between occurrence and reporting year (the reporting year is the year in which the loss is reported to the reinsurer), where  $lag_j^k = 0$  means that the two years coincide. Because the losses are known at the start of the current year, we must have  $lag_j^k \leq n - 1$  (no loss with a larger reporting lag occurring in the observation period  $k = 1, \dots, n$  would be known) and for each  $k = 1, \dots, n$ ,

$$k + lag_j^k \leq n$$

Each loss is assigned to a frequency as-if adjustment segment, where for every segment an exposure  $e_k^{seg}$  is given for each year  $k$  in the observation period and  $e_{CY}^{seg}$  for the exposure for the current year. The choice of the as-if adjustment segments and the exposure measures is related to the assumed underlying stochastic model explained in the following.

### Underlying stochastic model

We distinguish occurred losses from reported losses:

- Numbers of *occurred losses* are denoted by  $N_{\dots}$
- Numbers of *reported/known losses* are denoted by  $\tilde{N}_{\dots}$

We assume

- The loss occurrences for each segment  $seg$  follow an inhomogeneous (or nonhomogeneous) Poisson process on the real line with a time-dependent intensity function  $\lambda_{seg}(t) > 0$ .

More specifically, we consider successive years  $]k, k + 1]$  for  $k = 1, \dots, n$  and  $k = CY$  and assume that the number  $N_k^{seg}$  of losses/points in each year is Poisson distributed, independent between different years  $k$ , with mean given by (where the first equality holds generally for inhomogeneous Poisson processes and the second is specific):

$$E[N_k^{seg}] = \int_{]k, k+1]} \lambda_{seg}(t) dt = e_k^{seg} \cdot \lambda_{seg}$$

where  $e_k^{seg}$  is the exposure for the segment  $I_k$ , and  $\lambda_{seg} > 0$  is the annual frequency by unit of exposure for the segment. In particular, for the current year,

$$E[N_{CY}^{seg}] = e_{CY}^{seg} \cdot \lambda_{seg}$$

### Frequency as-if adjustment segments and factors

The above underlying stochastic model is linked to frequency as-if adjustment segments and factors. As an illustration, if  $e_k^{seg} > 0$ , we get for example

$$E[N_{CY}^{seg}] = \frac{e_{CY}^{seg}}{e_k^{seg}} \cdot E[N_k^{seg}]$$

i.e. for the frequency as-if adjustment segment  $seg$ , the frequency  $E[N_k^{seg}]$  for occurrence year  $k$  is transformed to the frequency for the current year by multiplication with the frequency as-if adjustment factor  $\frac{e_{CY}^{seg}}{e_k^{seg}}$ . If  $e_k^{seg} = 0$ , we assume that a priori  $N_k^{seg} = 0$ , i.e. there can be no losses if there is no (historical) exposure.

### Reporting lags and known losses

Because of the reporting lags, the available observations  $x_j^k$  from the observation period do not correspond to all losses  $N_k^{seg}$ , but only to the losses  $\tilde{N}_k^{seg}$  known at time  $t = 0$  (start of the current year). Analogously, we define the number  $\tilde{N}_{lag}^{seg}$  of known losses in the entire observation period to the segment  $seg$  with reporting lag  $lag$ . A historical loss  $x_j^k$  with reporting lag  $lag$  is known if and only if

$$k + lag \leq n$$

In particular, in any case,  $lag \leq n - 1$ .

We make the assumption:

- Assumption: There is no loss with  $lag \geq n$ .<sup>1</sup>

To express the link between known losses and reporting lags, we need to introduce the quantity  $N_{k,lag}^{seg}$  of all occurred losses to the segment  $seg$  in the occurrence year  $k$  with reporting lag  $lag$  and can then express the number  $\tilde{N}_k^{seg}$  of known losses with occurrence year  $k$  and segment  $seg$  by

$$\tilde{N}_k^{seg} = \sum_{lag=0}^{n-k} N_{k,lag}^{seg}$$

<sup>1</sup> This is akin to a "tail factor =1" assumption in a development triangle setting.

and the number  $\tilde{N}_{lag}^{seg}$  of known losses with reporting lag  $lag$  and segment  $seg$  by

$$\tilde{N}_{lag}^{seg} = \sum_{k=1}^{n-lag} N_{k,lag}^{seg}$$

For  $k = 1 \dots n - lag$ , the number  $N_{k,lag}^{seg}$  of occurred losses is known, i.e.  $\tilde{N}_{k,lag}^{seg} = N_{k,lag}^{seg}$ .

### Extension of underlying stochastic model to reporting lags

To be able to work with reporting lags, it is desirable to extend the underlying stochastic model to  $N_{k,lag}^{seg}$ , i.e. to assume that

$$E[N_{k,lag}^{seg}] = e_{k,lag}^{seg} \cdot \lambda_{seg}$$

with

$$\sum_{lag=0}^{n-1} e_{k,lag}^{seg} = e_k^{seg}$$

This is based on the

- Assumption: the same frequency rate  $\lambda_{seg}$  applies to losses assigned to the same (frequency as-if adjustment) segment  $seg$  with different reporting lags.

This can be justified through the definition of the segments: if there would be a difference between different reporting lags, the segments would have to be defined more granular to reflect this difference.

The problem with the above extension is that the exposure  $e_{k,lag}^{seg}$  per reporting lag is likely not available (and likely not reasonably definable). Given this, we make the

- Assumption: the ratio  $r_{lag}^{seg}$  between a given reporting lag exposure  $e_{k,lag}^{seg}$  and the total exposure  $e_k^{seg}$  is the same for each year  $k = 1, \dots, n$  and  $k = CY$ . I.e. there are numbers  $r_{lag}^{seg} \geq 0$  so that

$$e_{k,lag}^{seg} = r_{lag}^{seg} \cdot e_k^{seg}$$

It follows that for  $k = 1, \dots, n$  and  $k = CY$ ,

$$E[N_{k,lag}^{seg}] = e_k^{seg} \cdot r_{lag}^{seg} \cdot \lambda_{seg}$$

with

$$\sum_{lag=0}^{n-1} r_{lag}^{seg} = 1$$

By summing over all years  $k = 1, \dots, n - \text{lag}$ ,

$$E[\tilde{N}_{\text{lag}}^{\text{seg}}] = \left( \sum_{k=1}^{n-\text{lag}} e_k^{\text{seg}} \right) \cdot r_{\text{lag}}^{\text{seg}} \cdot \lambda_{\text{seg}}$$

### Estimate of the frequency for the current year

The objective is to find an estimate for the expected frequency for the current year

$$E[N_{\text{CY}}] = \sum_{\text{seg}} E[N_{\text{CY}}^{\text{seg}}] = \sum_{\text{seg}} \sum_{\text{lag}=0}^{n-1} E[N_{\text{CY},\text{lag}}^{\text{seg}}]$$

Using the expression above for  $E[N_{\text{CY},\text{lag}}^{\text{seg}}]$ , this can be written

$$E[N_{\text{CY}}] = \sum_{\text{seg}} \sum_{\text{lag}=0}^{n-1} e_{\text{CY}}^{\text{seg}} \cdot r_{\text{lag}}^{\text{seg}} \cdot \lambda_{\text{seg}}$$

Candidates for estimators  $\widehat{r_{\text{lag}}^{\text{seg}} \cdot \lambda_{\text{seg}}}$  in this expression are, using the formulas for  $E[N_{k,\text{lag}}^{\text{seg}}]$  and  $E[\tilde{N}_{\text{lag}}^{\text{seg}}]$  from above, for each of the years  $k = 1, \dots, n - \text{lag}$  with positive exposure ( $e_k^{\text{seg}} > 0$ ), replacing  $E[N_{k,\text{lag}}^{\text{seg}}]$  with  $N_{k,\text{lag}}^{\text{seg}}$ ,

$$\frac{N_{k,\text{lag}}^{\text{seg}}}{e_k^{\text{seg}}}$$

and for the sum over all years  $k = 1, \dots, n - \text{lag}$  (with positive exposure), provided  $\sum_{k=1}^{n-\text{lag}} e_k^{\text{seg}} \neq 0$ ,

$$\widehat{r_{\text{lag}}^{\text{seg}} \cdot \lambda_{\text{seg}}} = \frac{\tilde{N}_{\text{lag}}^{\text{seg}}}{\sum_{k=1}^{n-\text{lag}} e_k^{\text{seg}}}$$

Note that, if  $\sum_{k=1}^{n-\text{lag}} e_k^{\text{seg}} = 0$ , then also  $\tilde{N}_{\text{lag}}^{\text{seg}} = 0$ , as there can be no losses if there is no exposure. The above estimator corresponds to the weighted mean of the estimators from the individual years  $k$  using the weights

$$\frac{e_k^{\text{seg}}}{\sum_{l=1}^{n-\text{lag}} e_l^{\text{seg}}}$$

A justification for the selection of the weights is that the experience from years with higher exposure is considered to be more credible. One can also show that the above estimator is the maximum likelihood estimator (additionally assuming that  $N_{k,\text{lag}}^{\text{seg}}$  are Poisson distributed).

We then get the estimate  $Z$  of  $E[N_{\text{CY}}]$  as

$$Z = \sum_{seg} \sum_{\substack{lag=0 \\ \sum_{k=1}^{n-lag} e_k^{seg} \neq 0}}^{n-1} \frac{e_{CY}^{seg}}{\sum_{k=1}^{n-lag} e_k^{seg}} \cdot \tilde{N}_{lag}^{seg}$$

It is straightforward to verify that this estimator is unbiased, i.e.  $E[Z] = E[N_{CY}]$ . It can be written in terms of the experience scenarios  $s$  and their assigned segments  $seg(s)$  and reporting lags  $lag(s)$  as

$$Z = \sum_{\substack{\text{experience} \\ \text{scenario } s}} \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}}$$

### Justification for the frequency estimate

If  $\sum_{k=1}^{n-lag} e_k^{seg} \neq 0$ , the estimator  $\widehat{r_{lag}^{seg} \cdot \lambda_{seg}}$  is unbiased, as

$$E \left[ \widehat{r_{lag}^{seg} \cdot \lambda_{seg}} \right] = \frac{E[\tilde{N}_{lag}^{seg}]}{\sum_{k=1}^{n-lag} e_k^{seg}} = r_{lag}^{seg} \cdot \lambda_{seg}$$

For its variance we get, using that  $Var(\tilde{N}_{lag}^{seg}) = E[\tilde{N}_{lag}^{seg}]$  because of the Poisson assumption, and using the expression for  $E[\tilde{N}_{lag}^{seg}]$  from above,

$$Var \left( \widehat{r_{lag}^{seg} \cdot \lambda_{seg}} \right) = \frac{E[\tilde{N}_{lag}^{seg}]}{\left( \sum_{k=1}^{n-lag} e_k^{seg} \right)^2} = \frac{r_{lag}^{seg} \cdot \lambda_{seg}}{\sum_{k=1}^{n-lag} e_k^{seg}}$$

As further justification for the estimator, we show that the estimator  $\widehat{r_{lag}^{seg} \cdot \lambda_{seg}}$  is the minimum variance estimator (among unbiased estimators) in that its variance attains the Cramer-Rao lower bound. For unbiased estimators  $\widehat{\theta_{lag}^{seg}}$  of

$$\theta_{lag}^{seg} := r_{lag}^{seg} \cdot \lambda_{seg}$$

the Cramer-Rao lower bound is

$$Var \left( \widehat{\theta_{lag}^{seg}} \right) \geq \frac{1}{I \left( \widehat{\theta_{lag}^{seg}} \right)}$$

where

$$I \left( \widehat{\theta_{lag}^{seg}} \right) = E \left[ - \frac{\partial^2}{\partial \theta^2} \left( \log \left( f(X, \theta_{lag}^{seg}) \right) \right) \right]$$

where the expectation is taken over the observations  $X = \{N_{k,lag}^{seg}, k = 1, \dots, n - lag\}$ , and  $f(X, \theta_{lag}^{seg})$  is the likelihood of the observations  $X$  given the parameter  $\theta_{lag}^{seg}$ . For  $f(X, \theta_{lag}^{seg})$ , given that  $N_{k,lag}^{seg}$  are independent Poisson random variables with mean  $E[N_{k,lag}^{seg}] = e_k^{seg} \cdot \theta_{lag}^{seg}$ , we get

$$f(X, \theta_{lag}^{seg}) = \prod_{k=1}^{n-lag} e^{-e_k^{seg} \cdot \theta_{lag}^{seg}} \cdot \frac{(e_k^{seg} \cdot \theta_{lag}^{seg})^{N_{k,lag}^{seg}}}{(N_{k,lag}^{seg})!}$$

It follows that

$$\begin{aligned} -\frac{\partial^2}{\partial \theta^2} \left( \log(f(X, \theta_{lag}^{seg})) \right) &= -\frac{\partial^2}{\partial \theta^2} \left( \sum_{k=1}^{n-lag} -e_k^{seg} \cdot \theta_{lag}^{seg} + N_{k,lag}^{seg} \cdot (\log(e_k^{seg}) + \log(\theta_{lag}^{seg})) - \log((N_{k,lag}^{seg})!) \right) \\ &= \sum_{k=1}^{n-lag} \frac{N_{k,lag}^{seg}}{(\theta_{lag}^{seg})^2} \end{aligned}$$

and thus, using  $E[N_{k,lag}^{seg}] = e_k^{seg} \cdot \theta_{lag}^{seg}$ ,

$$I(\widehat{\theta_{lag}^{seg}}) = E \left[ \sum_{k=1}^{n-lag} \frac{N_{k,lag}^{seg}}{(\theta_{lag}^{seg})^2} \right] = \frac{\sum_{k=1}^{n-lag} e_k^{seg}}{\theta_{lag}^{seg}}$$

Consequently,  $\widehat{r_{lag}^{seg} \cdot \lambda_{seg}}$  is indeed the minimal variance estimator, as we get with the Cramer-Rao lower bound:

$$Var(\widehat{\theta_{lag}^{seg}}) \geq \frac{\theta_{lag}^{seg}}{\sum_{k=1}^{n-lag} e_k^{seg}} = Var(\widehat{r_{lag}^{seg} \cdot \lambda_{seg}})$$

### Uncertainty of the frequency estimate

For the estimation uncertainty of the frequency estimator  $Z$ , we estimate its variance. Due to independence of the random variables  $\tilde{N}_{lag}^{seg}$  and because  $Var(\tilde{N}_{lag}^{seg}) = E[\tilde{N}_{lag}^{seg}]$  because of the Poisson assumption, and using the expression for  $E[\tilde{N}_{lag}^{seg}]$  from above, we get

$$Var(Z) = \sum_{seg} \sum_{\substack{lag=0 \\ \sum_{k=1}^{n-lag} e_k^{seg} \neq 0}}^{n-1} \left( \frac{e_{CY}^{seg}}{\sum_{k=1}^{n-lag} e_k^{seg}} \right)^2 \cdot \left( \sum_{k=1}^{n-lag} e_k^{seg} \right) \cdot r_{lag}^{seg} \cdot \lambda_{seg}$$

Replacing  $r_{lag}^{seg} \cdot \lambda_{seg}$  by its estimator  $\widehat{r_{lag}^{seg} \cdot \lambda_{seg}}$  from above, we get an estimator  $\widehat{Var(Z)}$  for the variance

$$\begin{aligned} \widehat{Var(Z)} &= \sum_{seg} \sum_{\substack{lag=0 \\ \sum_{k=1}^{n-lag} e_k^{seg} \neq 0}}^{n-1} \left( \frac{e_{CY}^{seg}}{\sum_{k=1}^{n-lag} e_k^{seg}} \right)^2 \cdot \tilde{N}_{lag}^{seg} \\ &= \sum_{\substack{experience \\ scenario s}} \left( \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}} \right)^2 \end{aligned}$$

### Estimated frequency including estimation uncertainty uplift for the current year

We can set the estimated frequency  $\lambda_{IE1}$  including the estimation uncertainty uplift for the current year to the sum of the frequency estimate and the standard deviation estimate derived from the variance estimate:

$$\lambda_{IE1} = \sum_{\substack{\text{experience} \\ \text{scenario } s}} \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}} + \sqrt{\sum_{\substack{\text{experience} \\ \text{scenario } s}} \left( \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}} \right)^2}$$

The objective is:

- $\lambda_{IE1}$  should be at least as large as the true frequency (from the inhomogeneous Poisson process) in sufficiently many cases.

An alternative approach, in which the estimated frequency does not include the estimation uncertainty uplift, but this is considered in the volatility, is presented in Section 3.3.

### Simulation case study

For the benchmark model, we assume that the underlying stochastic model is known and calculate the true frequency  $E[N_{CY}]$  analytically.

The estimated frequency  $\lambda_{IE1}$  (incl. uplift) is then calculated independently 1'000 times by drawing one realization from every Poisson random variable  $N_{k,lag}^{seg}$  and using the above formulas.

The percentage of draws for which the estimated frequency  $\lambda_{IE1}$  exceeds the true frequency  $E[N_{CY}]$  is reported in the table below for different values of the overall historical exposure  $\sum_{k=1}^5 e_k$  for an observation period of five years (all other parameters remaining fixed).

$e_{CY}$	47	47	47	47
$\sum_{k=1}^5 e_k$	2.55	25.5	255.5	2555
$\frac{\#\{\lambda_{IE1}^{trial} > E[N_{CY}], \text{ trial} = 1, \dots, 1000\}}{1000}$	43.5%	73.5%	80.1%	82.5%

The percentages shown above are the probabilities that

$$Z + \sqrt{\text{Var}(Z)} \geq E[N_{CY}]$$

where  $Z$  is the estimator of  $E[N_{CY}]$  and  $\widehat{Var}(Z)$  is the estimator of the variance of  $Z$ . Denoting by  $\Phi$  the cumulative distribution function of the standard normal distribution, and assuming that  $Z$  is normal distributed, the probability that

$$Z + \sqrt{\widehat{Var}(Z)} \geq E[N_{CY}]$$

is close to and larger than 84%, because  $\Phi^{-1}(0.84)$  is less than but close to 1 and

$$\begin{aligned} P\left[Z + \sqrt{\widehat{Var}(Z)}\Phi^{-1}(0.84) \geq E[N_{CY}]\right] &= P\left[Z \geq E[N_{CY}] - \sqrt{\widehat{Var}(Z)}\Phi^{-1}(0.84)\right] \\ &= P\left[Z \geq E[N_{CY}] + \sqrt{\widehat{Var}(Z)}\Phi^{-1}(0.16)\right] = 1 - P\left[\frac{Z - E[N_{CY}]}{\sqrt{\widehat{Var}(Z)}} \leq \Phi^{-1}(0.16)\right] = 0.84 \end{aligned}$$

The numbers in the table above deviate from 84% because  $Z$  is only approximately normal distributed and because  $\widehat{Var}(Z)$  is only approximately equal to  $Var(Z)$ . If the realizations  $\tilde{N}_{lag}^{seg}$  are smaller than their means  $E[\tilde{N}_{lag}^{seg}]$ , then both the estimator of  $E[N_{CY}]$  and the variance are underestimated. In addition, there may be an estimation error from the limited number of 1'000 trials.

We can see that the deviations between 84% and the simulated numbers from the table reduce as the historical exposure increases, i.e. as "more historical experience" is available, corresponding to higher values of  $E[\tilde{N}_{lag}^{seg}]$ . The explanation for this is as follows. If the means  $E[\tilde{N}_{lag}^{seg}]$  are close to zero, the probability to underestimate the "true" expected frequency can be quite high, as  $P(N_{lag}^{seg} = 0, \forall seg, \forall lag) = \exp(-\sum E[N_{lag}^{seg}])$  can get close to 1 for small  $E[N_{lag}^{seg}]$ . This leads with high probability to an estimator of zero (even with the uplift), because most realizations will be zero. However, in reality, this would translate to a situation in which there are almost no historical large event losses, which would have to be ameliorated by lower IE1 modelling thresholds.

### As-if adjusted frequencies incl. IBNyR of experience scenarios

We would like to assign to each experience scenario  $s$  a frequency  $f_s$  so that the sum over all experience scenarios of this frequency is equal to the total estimated frequency  $\lambda_{IE1}$  including estimation uncertainty uplift from above. We do this by first determining a frequency  $f'_s$  that does not consider the uplift and then scaling up proportionally, i.e.

$$f_s = f'_s \cdot \frac{\lambda_{IE1}}{\sum_{\text{experience scenario } s} \left( \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}} \right)}$$

The estimated frequency  $f'_s$  is intended to be as-if adjusted to the current year and including consideration of IBNyR. The natural selection in view of the estimator  $Z$  is

$$f'_s = \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}}$$

With this selection, the sum over all experience scenarios of  $f'_s$  is equal to  $Z$ , and the frequency is as-if adjusted to the current year and considers IBNyR in the following sense: assume that we would disregard

IBNyR, i.e. disregard the fact that losses may have occurred but not yet be known because of the reporting lag. In this case, we would consider the whole exposure of the observation period  $k = 1, \dots, n$  for the as-if adjustment, so the frequency estimate  $\bar{f}'_s$  disregarding IBNyR would be

$$\bar{f}'_s = \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^n e_k^{seg(s)}}$$

For  $\sum_{k=1}^{n-lag} e_k^{seg} > 0$ , we can interpret the difference  $f'_s - \bar{f}'_s$  given by

$$\tilde{f}_{IBNyR}(seg, lag) = \frac{e_{CY}^{seg}}{\sum_{k=1}^{n-lag} e_k^{seg}} - \frac{e_{CY}^{seg}}{\sum_{k=1}^n e_k^{seg}}$$

as IBNyR that could be used to produce "artificial losses" for each of the years  $n - lag + 1, \dots, n$  for which information on occurred losses may be lacking due to IBNyR. Artificial losses can be assigned only to those years  $k = n - lag + 1, \dots, n$  for which the corresponding exposure  $e_k^{seg}$  is positive (because in years with no exposure there can be no losses). We denote the number of such years by:

$$b(seg, lag) = \#\{k = n - lag + 1, \dots, n | e_k^{seg} > 0\} \leq lag$$

The IBNyR frequency  $\tilde{f}_{IBNyR}(seg, lag)$  can be allocated to each such year  $k = n - lag + 1, \dots, n$  with positive exposure  $e_k^{seg}$  in equal portions, producing for each such year an artificial loss with frequency (if  $b(seg, lag) \neq 0$ )

$$\frac{\tilde{f}_{IBNyR}(seg, lag)}{b(seg, lag)}$$

Note that whenever there is IBNyR, i.e.  $\tilde{f}_j^{k, IBNyR}$  is positive, there is a year  $k = n - lag + 1, \dots, n$  to which the IBNyR can be allocated as its exposure is positive (otherwise we would have  $\sum_{k=1}^{n-lag} e_k^{seg} = \sum_{k=1}^n e_k^{seg}$ , i.e.  $\tilde{f}_j^{k, IBNyR} = 0$ ). Hence, whenever the above denominator is zero, the numerator is also zero. This procedure leads to additional "artificial" experience scenarios, but the problem is what severity to assign to these scenarios. We make the simplifying assumption:

- The severity of the additional "artificial" experience scenarios is the same as the (as-if adjusted) severity of the historical loss that has "produced" them.

With this assumption, no new scenarios must be produced, as for any original experience scenario, the sum of the corresponding "frequency without IBNyR" and IBNyR frequency can be assigned as total frequency to that experience scenario, ending up with the scenario frequency  $f'_s$  we have started with.

### As-if adjusted frequencies old vs new

Disregarding IBNyR/the reporting lags and the frequency uplift for the moment, the as-if adjusted frequency of an individual experience scenario we use is

$$\bar{f}'_s = \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^n e_k^{seg(s)}}$$

This differs from the formula provided in the "field test 2016"-version of the model description, where for scenario  $s$  with occurrence year  $k$ , it would be

$$\bar{f}'_s = \frac{e_{CY}^{seg(s)}}{e_k^{seg(s)}} \cdot \frac{1}{n}$$

The two formulas coincide if the exposures  $e_k^{seg(s)}$  are the same for all years in the observation period.

As an argument for the former over the latter formula, assume for the moment that there is only one loss  $x_1^l$  in the observation period, so that the corresponding  $\tilde{f}_1^l$  is the estimated expected loss frequency for the current year. In the former as opposed to the latter formula, this estimate also depends on the exposures for the years  $k \neq l$  in the observation period, and this is relevant for the estimation of the frequency, in this specific case because it contains the information that no loss has been observed in the years  $k \neq l$ .

To see why it is relevant, consider more concretely two special cases, where for the first case we assume that  $e_k^{seg(s)} = 0$  for  $k \neq l$  and for the second that  $e_k^{seg(s)} = e_l^{seg(s)}$  for  $k \neq l$ . The frequency estimate from the latter formula does not distinguish between the two cases. But in the first case there could not be any losses for  $k \neq l$ , whereas the fact that there were none for  $k \neq l$  in the second case is relevant and should decrease the frequency estimate.

The frequency we actually use considers IBNyR by not summing up to year  $n$ , but only up to year  $n - lag$ :

$$f'_s = \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}}$$

The reason for this is that a loss with a positive reporting lag  $lag > 0$  that had occurred in any of the years  $n - lag + 1, \dots, n$  would not be known at the start of the current year. Hence, in our example above, the fact that no corresponding loss has been observed in these years is not relevant (as it could not have been observed), so the exposures of those years should not be considered in the as-if adjustment of the frequency.

### 3.3 Frequency distribution if mean is random, e.g. Gamma

#### Frequency distribution

We consider a frequency random variable  $N$  that depends on a random variable  $\Lambda$  corresponding to its mean, so that

- $N|(\Lambda = \lambda)$  is Poisson distributed for any  $\lambda$  with mean  $E[N|\Lambda] = \Lambda$ .

Then:

$$E[N] = E[\Lambda], \quad Var(N) = E[\Lambda] + Var(\Lambda)$$

i.e.

$$\frac{Var(N)}{E[N]} = 1 + \frac{Var(\Lambda)}{E[\Lambda]}$$

This follows because, by assumption,  $E[N|\Lambda] = \Lambda$ , so  $E[N] = E[E[N|\Lambda]] = E[\Lambda]$ , and  $Var(N|\Lambda) = E[N|\Lambda]$  because of the Poisson assumption, so  $E[N^2|\Lambda] = Var(N|\Lambda) + E[N|\Lambda]^2 = \Lambda + \Lambda^2$ , so

$$Var(N) = E[E[N^2|\Lambda]] - E[E[N|\Lambda]]^2 = E[\Lambda] + E[\Lambda^2] - E[\Lambda]^2$$

If in addition:

- $\Lambda$  is Gamma-distributed with  $E[\Lambda] = \frac{\alpha}{\beta}$  and  $Var(\Lambda) = \frac{\alpha}{\beta^2}$

then  $N$  is negative Binomial distributed (well known) with

$$\frac{Var(N)}{E[N]} = 1 + \frac{1}{\beta}$$

### Parametrization

The above results can be combined with the results from Section 3.2 to derive an alternative parameterization of the frequency  $N$ :

$$E[N] = \sum_{\substack{\text{experience} \\ \text{scenario } s}} \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}}$$

$$Var(N) = \sum_{\substack{\text{experience} \\ \text{scenario } s}} \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}} + \sum_{\substack{\text{experience} \\ \text{scenario } s}} \left( \frac{e_{CY}^{seg(s)}}{\sum_{k=1}^{n-lag(s)} e_k^{seg(s)}} \right)^2$$

This can for example be used for calibrating a negative binomial distribution. The interpretation is that we take as the mean frequency the mean without any adjustment for estimation uncertainty, and consider the estimation uncertainty in the variance of the frequency.

### 3.4 As-if adjustments for experience scenarios

Experience scenarios are derived by as-if adjusting historical event losses. In the default approach, as-if adjustments are applied to event losses to reinsurer assigned to different as-if segments. For every historical event loss in a given segment,

- (1) the as-if adjustment for the severity consists in multiplying the event loss with a factor specific to the segment, and
- (2) for the frequency, multiplying the occurrence frequency by a factor specific to the segment.

In particular, in the default approach, the as-if adjusted historical losses to reinsurer are not calculated by applying the applicable current inward reinsurance structure to the as-if adjusted losses before inward reinsurance.

This is a disadvantage given the possible changes in inward reinsurance structures over time and because the impact of the current inward reinsurance structures is approximated by multiplicative as-if adjustment factors applied after application of the historical inward reinsurance structure. However, it is considered necessary for the default approach for the following reasons:

- (1) An as-if adjusted historical event loss in a segment is considered to be a representative for all possible event losses for the segment in the current year. A segment typically does not just contain one inward reinsurance contract.
- (2) The application of the current inward reinsurance structure to losses before inward reinsurance can lead to an underestimation of the occurrence frequencies, as there can be losses that were not reported under the prior year inward reinsurance structure but that would be relevant under the current structure.
- (3) It cannot be ruled out that the inward reinsurance underwriting over time is impacted by the loss experience, which can introduce a bias.

These points apply also more generally and imply that the as-if adjustments cannot be applied to a granularity that is too fine, as this could lead to an underestimation of losses.

However, under the conditions specified in Section 4.6 in the StandRe model description, it is possible to apply as-if adjustments to historical "losses to cedant" and then apply the current outward retrocession structure. Because there are only few inward reinsurance contracts, the effort for this approach is considerably lower than for a large number of contracts.

The alternative is possible because with respect to the above points:

- (1) An as-if adjusted historical event loss in an as-if adjustment segment can be considered as a representative for all possible event losses in the current year from that segment, as the segments are quite large.

- (2) The event losses considered are all event losses to the cedant that exceeded a given reporting threshold. Therefore, the problem with the underestimation does not apply.
- (3) The issue with the potential bias from underwriting based on loss experience, e.g. inward reinsurance contracts are not renewed when they have incurred large losses, may not be relevant as there are only few contracts.

### 3.5 Adjusting the frequencies of experience scenarios for non-experience scenarios

We explain the derivation of the formula for  $f'_s$  from Section 6.7.2 of model description v6.0:

$$f_s = f'_s \cdot \frac{\lambda_{IE1} - f_{nonexp}}{\lambda_{IE1}}$$

where  $f_{nonexp}$  is the sum of the expected occurrence frequencies of the non-experience scenarios

$$f_{nonexp} = \sum_{IE1 \text{ non-experience scenario } s} f_s$$

To this end, recall that the sum of the expected occurrence frequencies  $f'_s$  of the experience scenarios is equal to  $\lambda_{IE1}$ . To remove overlaps with the non-experience scenarios, this sum should be reduced to  $\lambda_{IE1}$  minus the sum  $f_{nonexp}$  of the expected occurrence frequencies of the non-experience scenarios. To achieve this, all expected occurrence frequencies  $f_s$  of the experience scenarios are scaled with the same factor. Hence the sum of the changed expected occurrence frequencies  $f'_s$  of the experience scenarios should be equal to  $\lambda_{IE1} - f_{nonexp}$ . The corresponding sum over the  $f'_s$  instead of  $f_s$  is equal to  $\lambda_{IE1}$ . From this, the above formula for  $f'_s$  follows.

The approach is thus based on the following assumptions:

- (1) In the default case, it is assumed that the non-experience scenarios do not increase the expected excess frequency  $\lambda_{IE1}$  estimated from the experience scenarios.
- (2) The frequency selection is applied proportionally to all experience-scenarios.

The default assumption behind (1) is that the experience scenarios “cover” the expected excess frequency at the modeling threshold, so “adding” a non-experience scenario does not change (increase) the expected excess frequency at the modeling threshold. This implies that the expected occurrence frequencies of the experience scenarios need to be reduced. (2) corresponds to the assumption that there is no sub-set of experience scenarios which corresponds to the non-experience scenarios.



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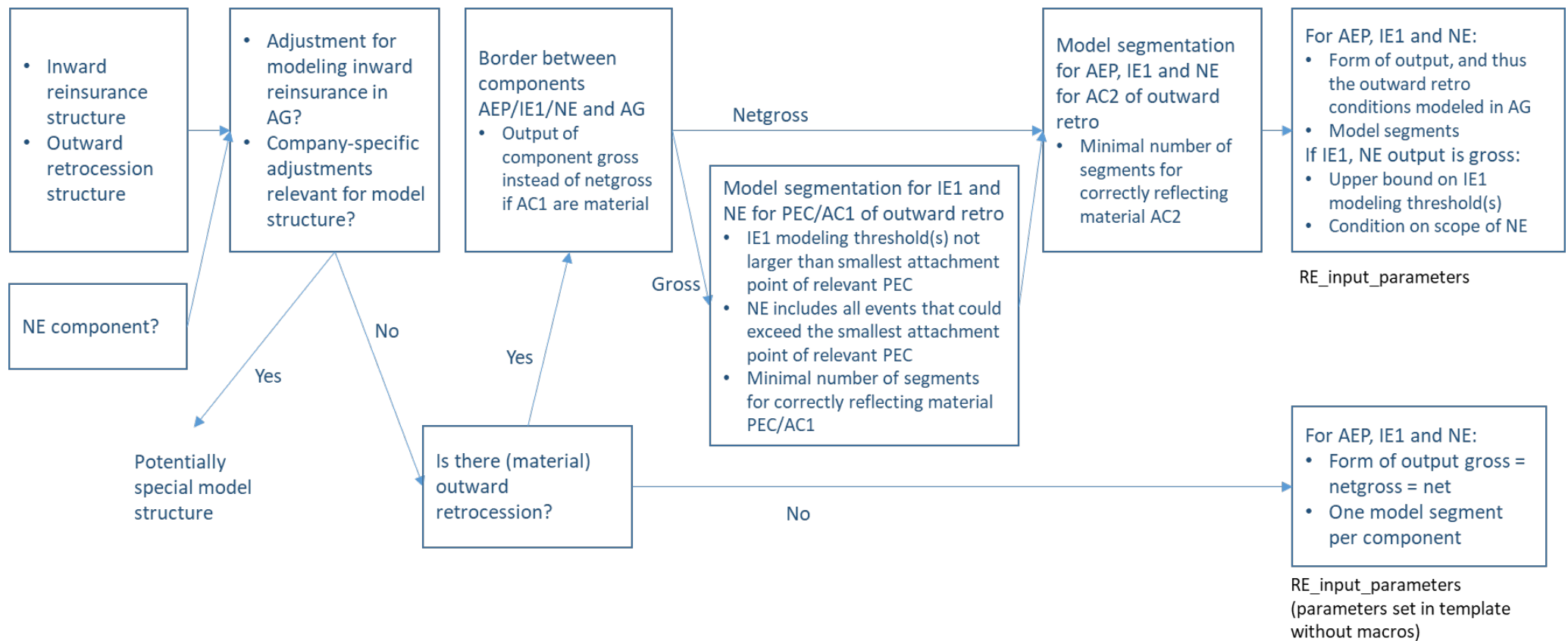
## 4 Charts – Overview of StandRe items



## Selection of model structure

Input

Output

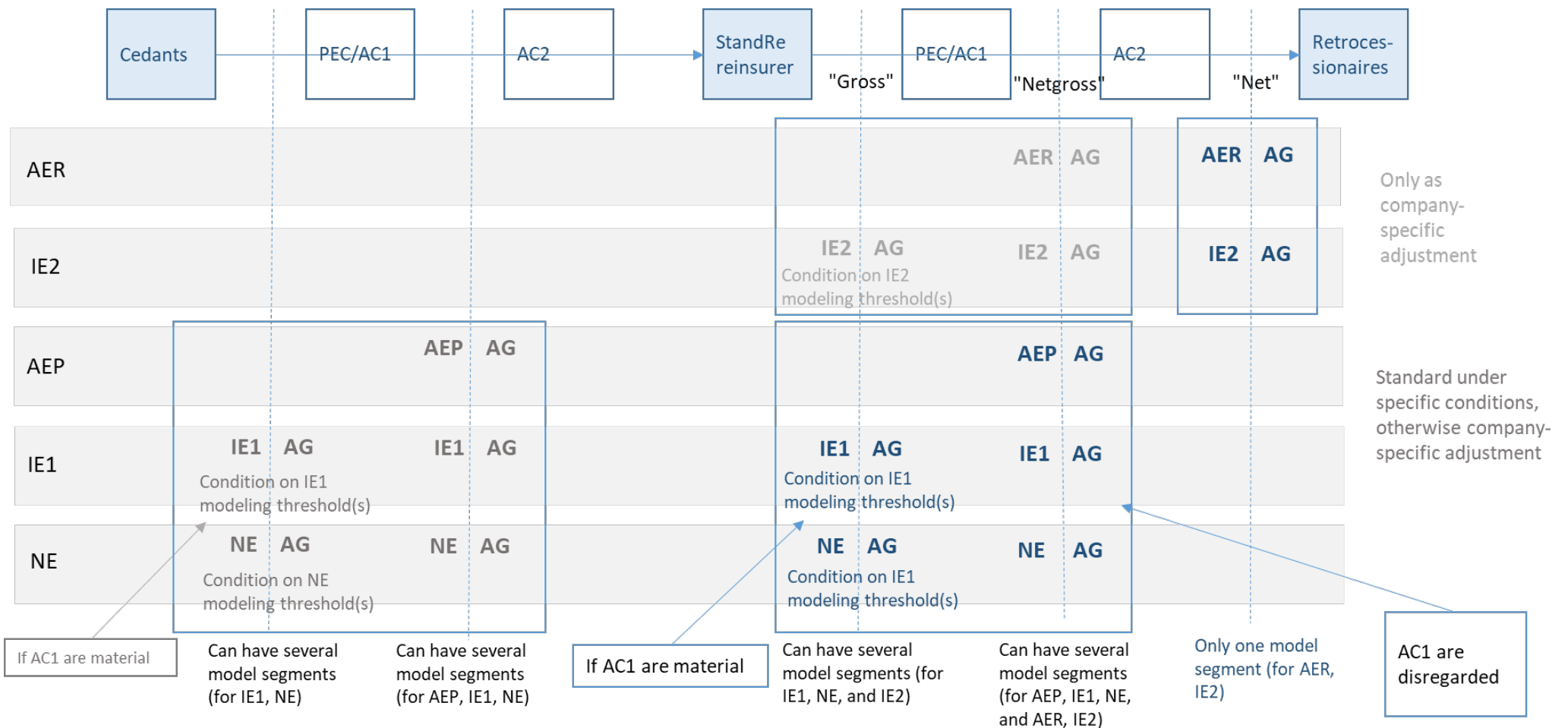


## Cession structure and StandRe components

RE\_input\_parameters

For each StandRe component (AER, IE2, AEP, IE1, NE), select exactly one separation point:

- everything to the left of the separation point is modeled in the component
- everything to the right of the separation point is modeled in AG



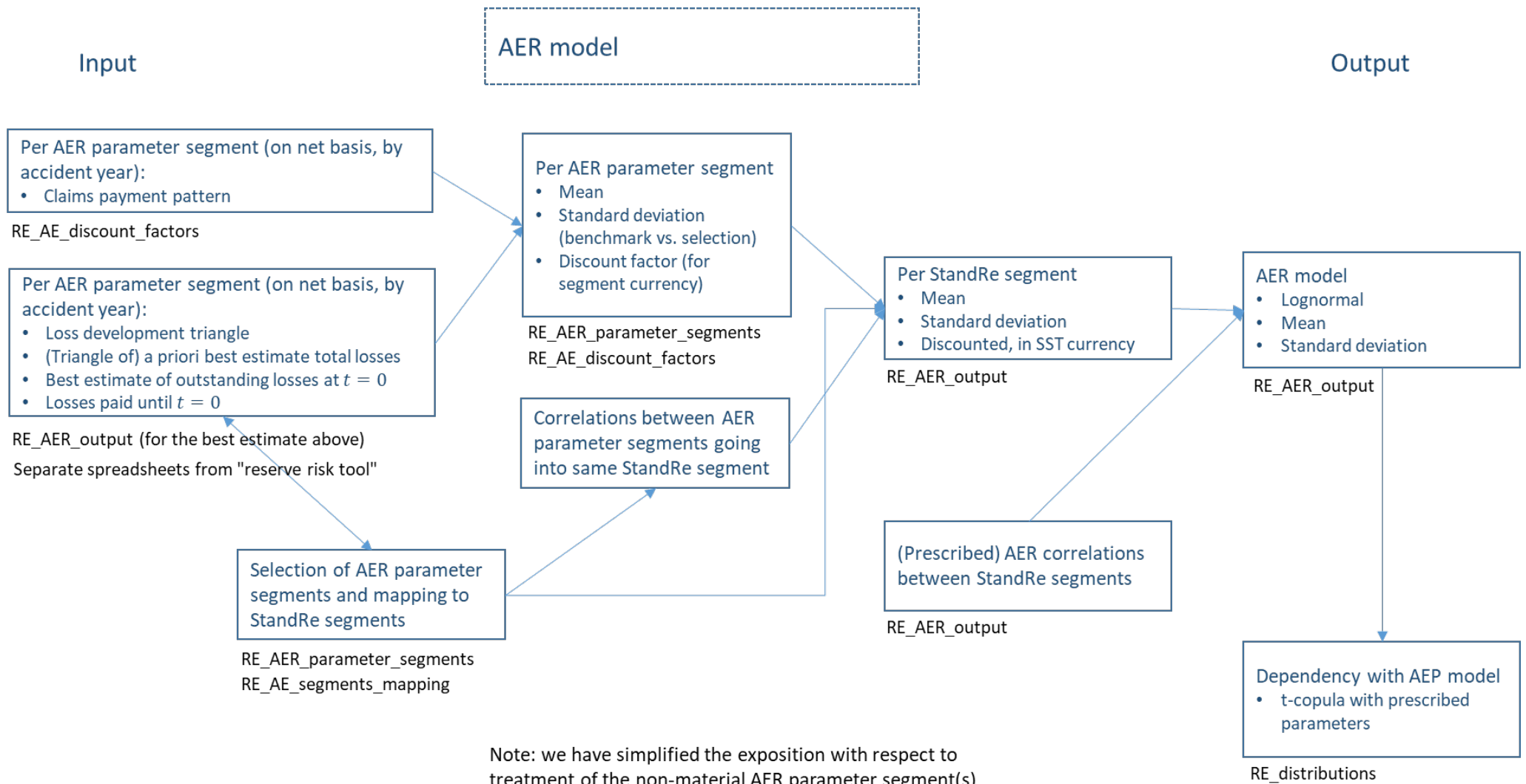
## Model segments - example

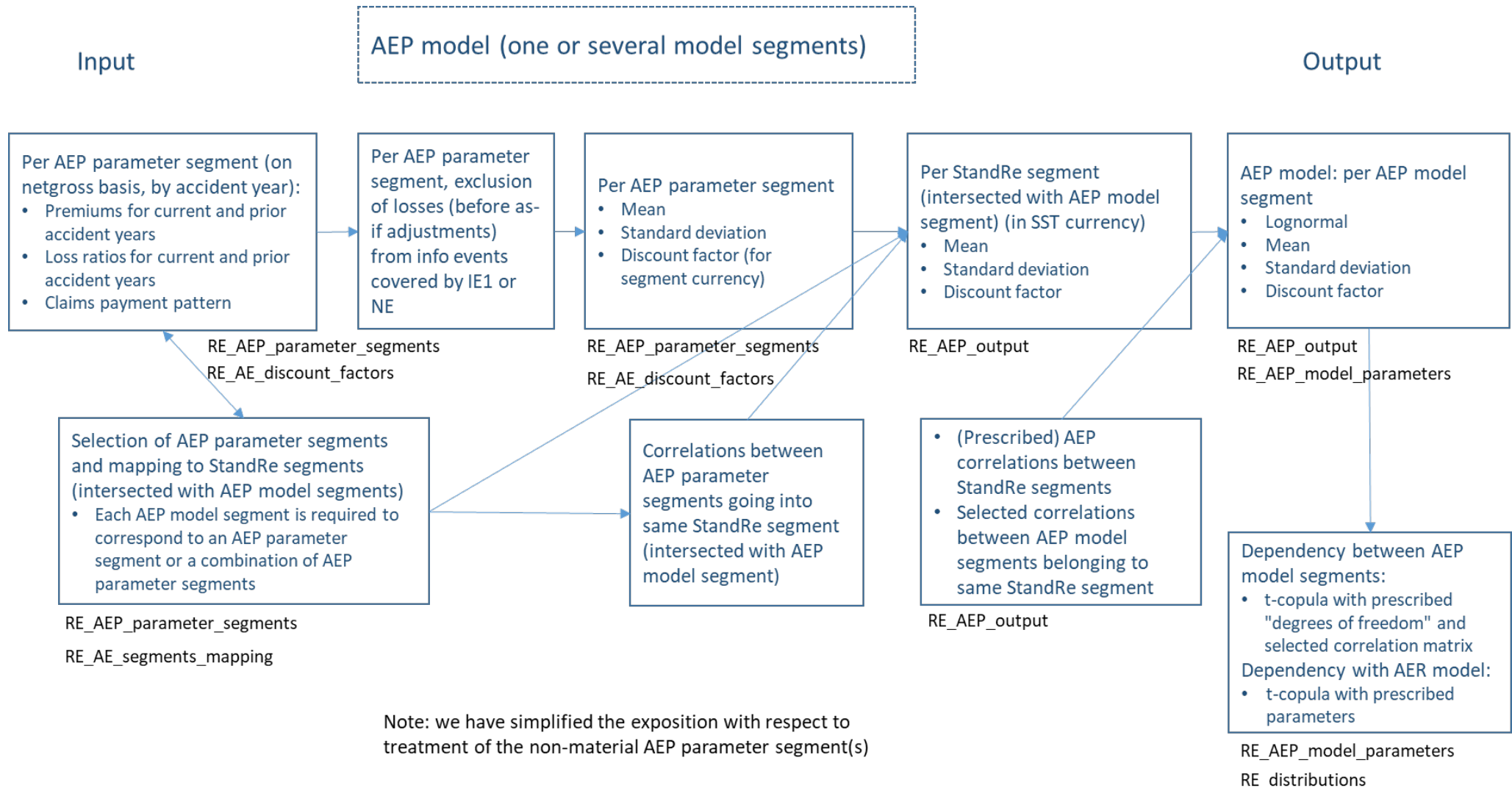
Here we assume that all PEC/AC1 are applied before the AC2 and all conditions are explicitly modeled in AG. The base segments below (e.g. different LOBs) are assumed to cover the whole portfolio. If e.g. XoL layers 2, 3 would not be modeled in AG, then for IE1 and NE, model segments 2 and 3 could be combined.

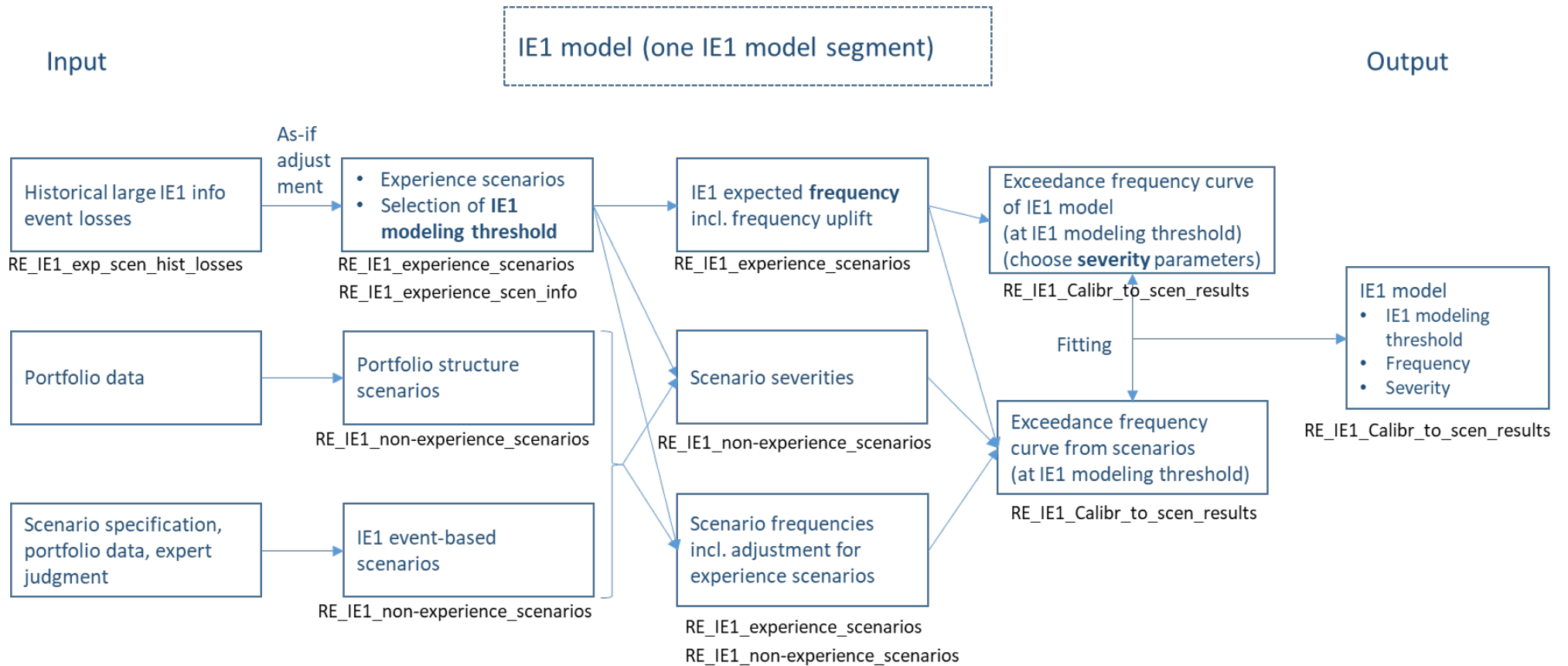
RE\_input\_parameters

	Base segment (BS) 1	Base segment 2	Base segment 3	Base segment 4	Base segment 5
Outward retrocession structure	PEC/AC1 <ul style="list-style-type: none"> <li>XoL layer 1 for BS1</li> </ul> AC2 <ul style="list-style-type: none"> <li>Stop Loss 1 for BS1</li> </ul>	PEC/AC1 <ul style="list-style-type: none"> <li>XoL layer 2 for BS2</li> </ul> AC2 <ul style="list-style-type: none"> <li>Stop Loss 2 for BS2 together with BS3</li> </ul>	PEC/AC1 <ul style="list-style-type: none"> <li>XoL layer 3 for BS3</li> </ul> AC2 <ul style="list-style-type: none"> <li>Stop Loss 2 for BS3 together with BS2</li> </ul>	PEC/AC1 <ul style="list-style-type: none"> <li>XoL layer 4 for BS4 Nat Cat only</li> </ul> AC2 <ul style="list-style-type: none"> <li>Stop Loss 3 for BS4 together with BS5</li> </ul>	PEC/AC1 <ul style="list-style-type: none"> <li>XoL layer 5 for BS5 Nat Cat only</li> </ul> AC2 <ul style="list-style-type: none"> <li>Stop Loss 3 for BS5 together with BS4</li> </ul>
AEP	AEP model segment 1	AEP model segment 2		AEP model segment 3	
IE1	IE1 model segment 1	IE1 model segment 2	IE1 model segment 3	IE1 model segment 4	
NE	NE model segment 1	NE model segment 2	NE model segment 3	NE model segment 4	NE model segment 5

This part of the model segmentation would not change if there were no or the same PEC/AC1 for BS1 and BS2, but still two different Stop Losses.



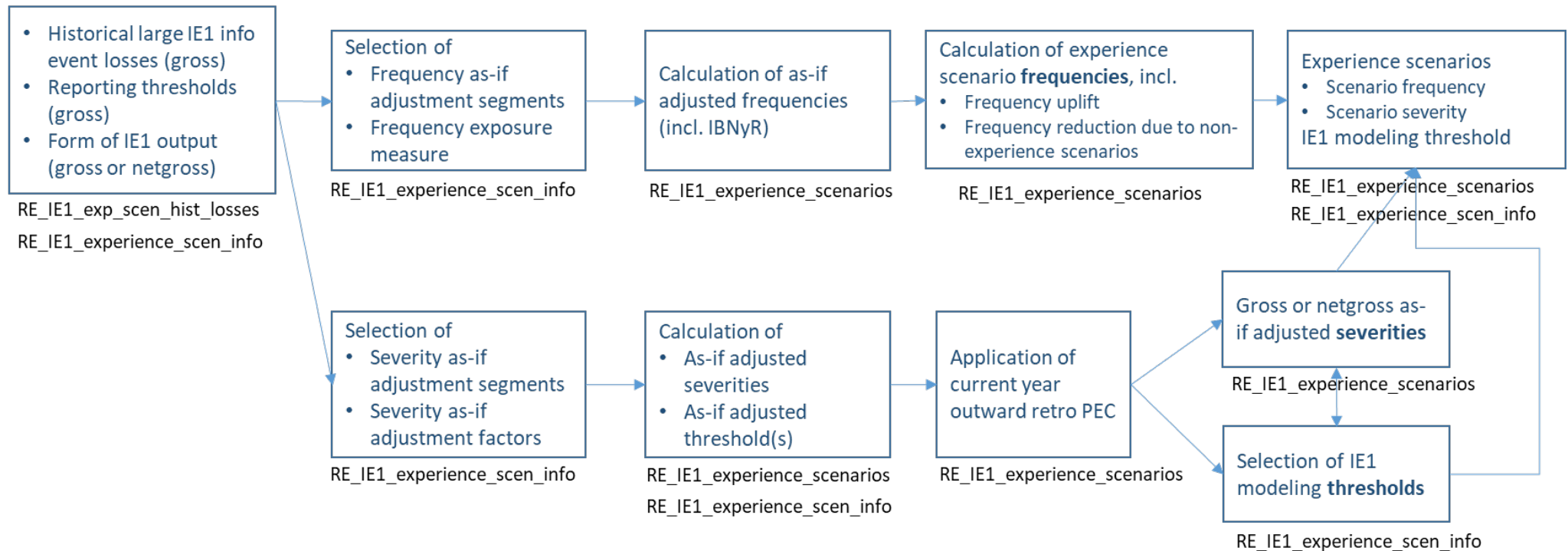


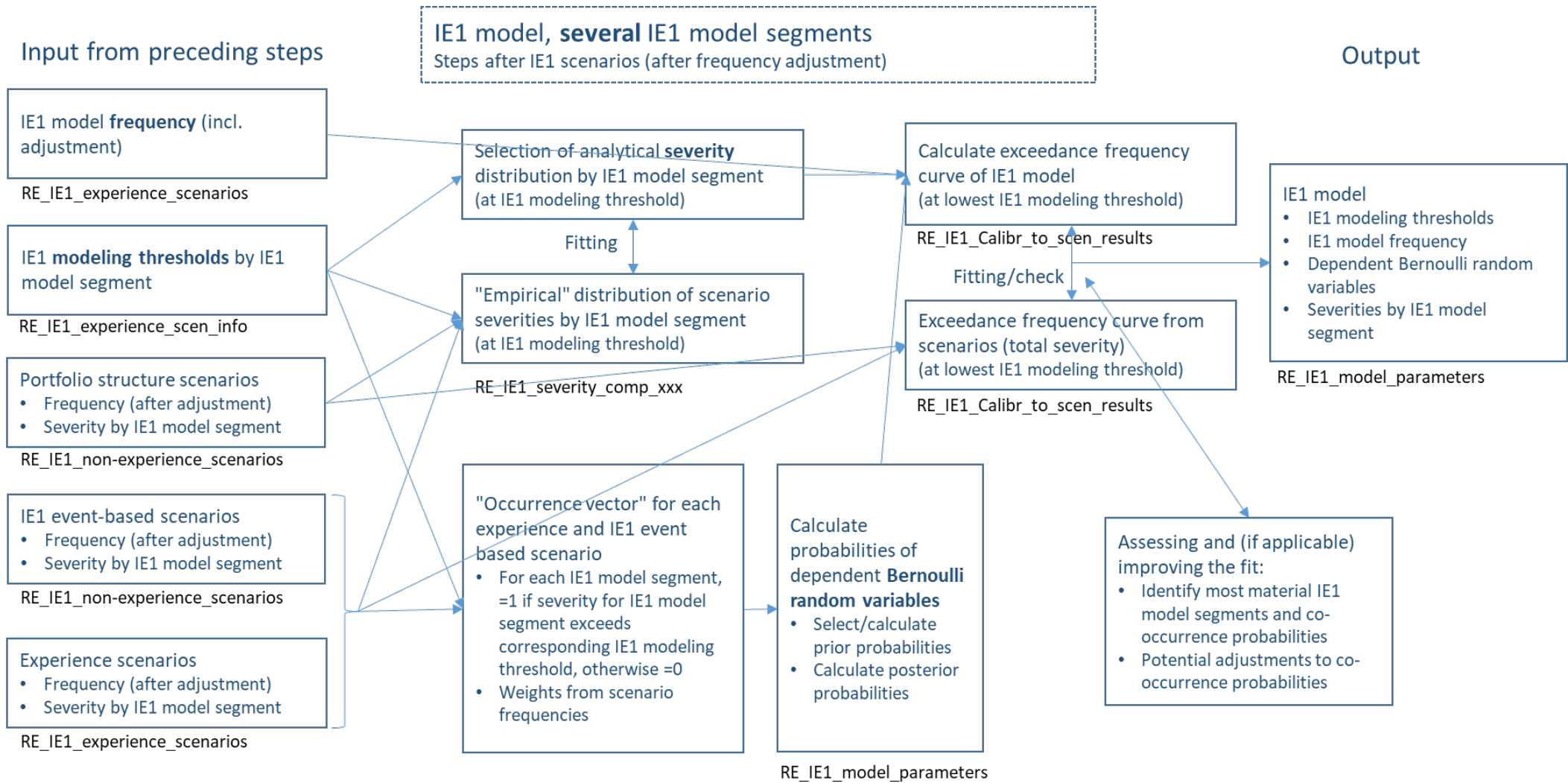


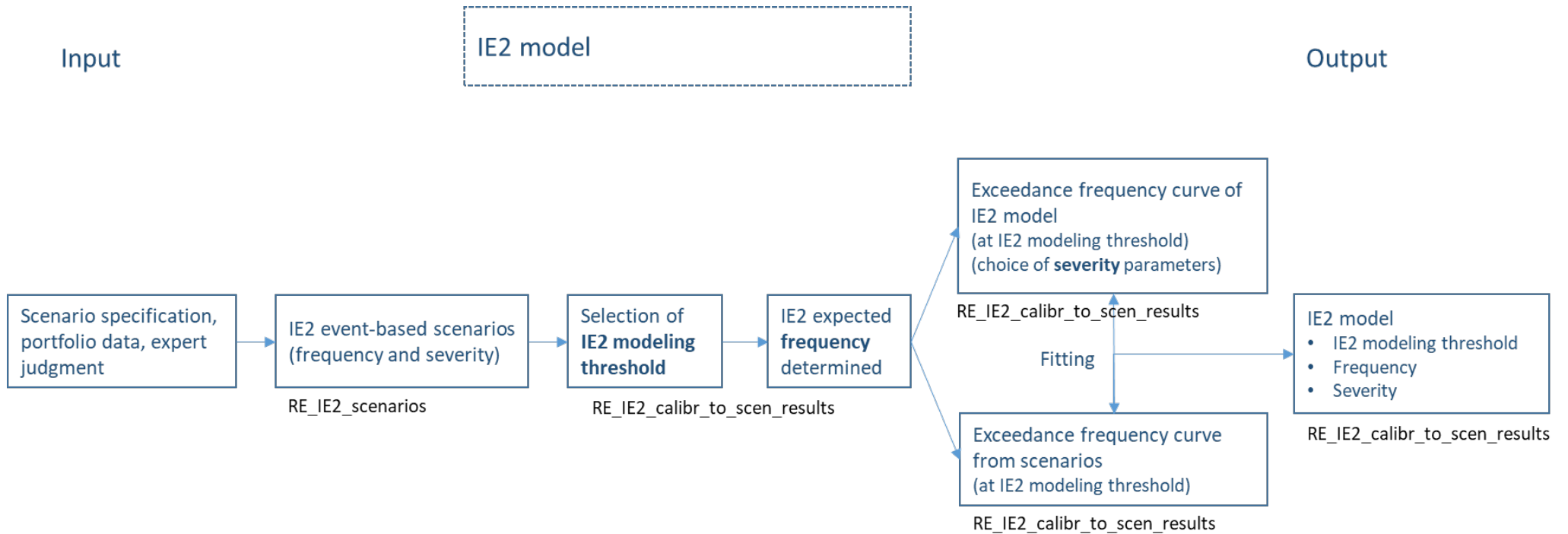
## Input

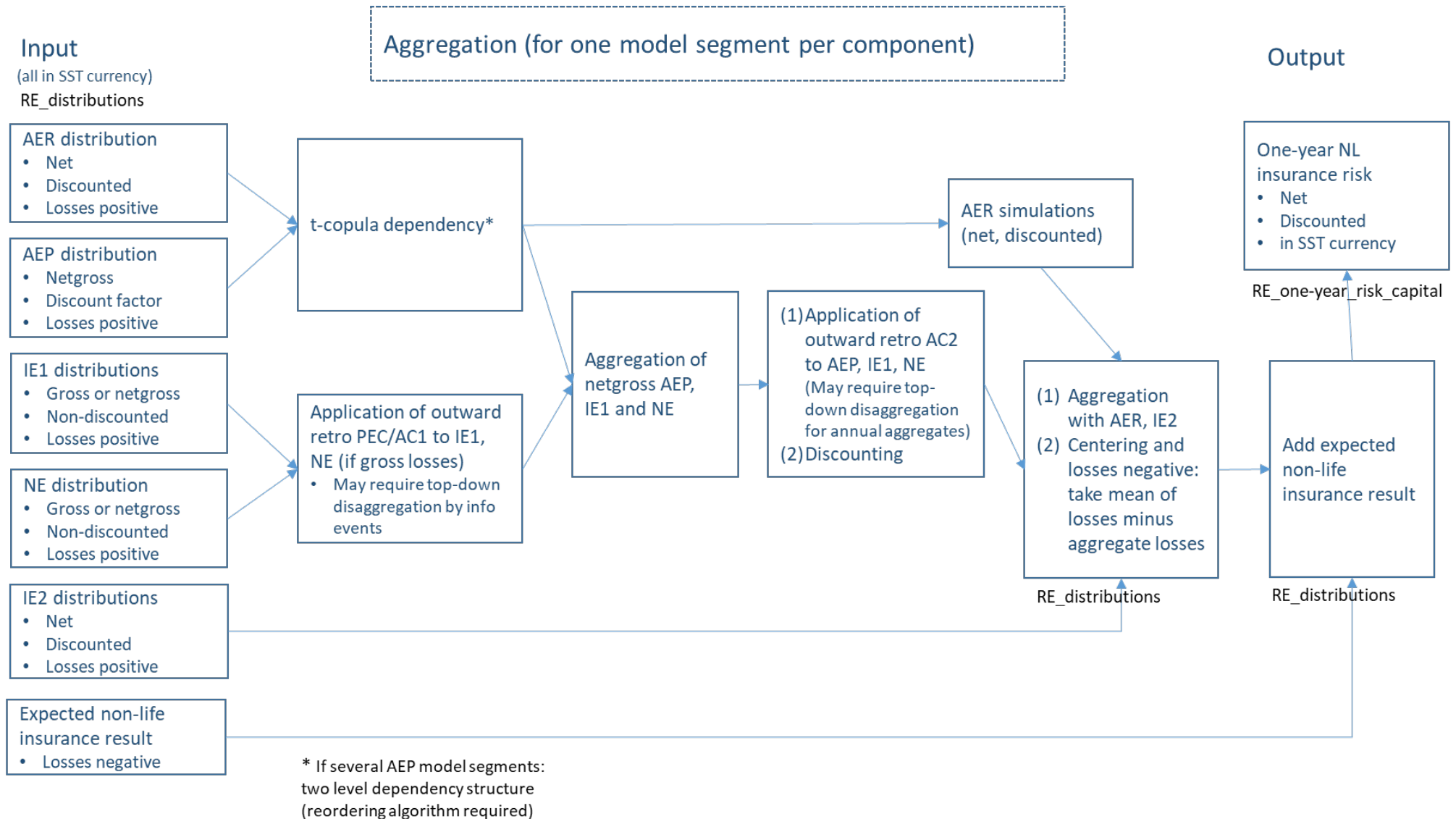
IE1 experience scenarios (one IE1 model segment)

## Output











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